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MEASUREMENT OF THE VELOCITY OF LIGHT BETWEEN MOUNT WILSON AND MOUNT SAN ANTONIO[1]

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ABSTRACT

The following is a continuation of the experiments described in the *Astrophysical Journal*, **60**, 256, 1924. The arrangement of apparatus differs slightly from that of the former investigation, allowing a more nearly normal incidence on the facets of the revolving mirror, and providing greater symmetry as well as increase in illumination.

Five independent series of measurements made with different revolving mirrors (one of steel having the form of a prism with eight facets, and another with twelve, and three of glass with eight, twelve, and sixteen facets) gave results showing a remarkable agreement.

The final result for the velocity of light *in vacuo* is 299,796 km per second.

The increasing appreciation of the importance of this fundamental constant of nature and the anticipation of the possibility of attaining a considerable increase in accuracy in its measurement justified the acceptance of the generous invitation of Dr. G. E. Hale, then director of the Mount Wilson Observatory, to make use of the facilities there available. It is with sincere appreciation that I take this opportunity of extending my acknowledgment of the many courtesies received and of the cordial co-operation of Dr. Hale and subsequently of Dr. Adams and the entire staff of the Observatory.

The result of the preliminary experiments described in a previ-

[1] *Contribution from the Mount Wilson Observatory*, No. 329.

ous article[1] gave for the final value of the velocity of light in air 299,735 km per second.

The correction[2] to *vacuum* is given by

$$n-1=(n_0-1)\frac{d}{d_0}=(n_0-1)\frac{p}{p_0}\frac{T_0}{T}$$

$$\frac{p}{p_0}=.822$$

and

$$\frac{T_0}{T}=.932\,,$$

whence

$$n-1=.000294\times.765$$

or

$$n=1.000225\,,$$

giving 67 km for the correction.[3]

Accordingly, for the preliminary determination

$$V=299{,}735+67$$

or

$$V=299{,}802\,.$$

A second series of observations with the glass octagon was begun in July of 1925, the arrangement of apparatus being essentially the same as in the preliminary work, except that instead of driving the electric fork ($N=132$) by make and break between platinum points, the fork ($N=528$) was driven by a vacuum-tube circuit, giving a rate far more nearly constant.

This rate was measured by comparison with a free pendulum furnished by the United States Coast and Geodetic Survey, as in the previous work; but with an important improvement in the stroboscopic method. This consists in allowing the light from a very narrow slit to fall on a mirror attached to the pendulum, forming, by means of a good achromatic lens, an image of the slit in the plane of an edge of the fork, where it is observed by an eyepiece.

[1] *Astrophysical Journal*, **60**, 256, 1924.

[2] The correction should be applied to the individual observations; but the result is not appreciably altered by taking the mean values given above.

[3] This was erroneously given as 85 km in the article cited.

This method of making the stroboscopic comparison was found to be far more convenient and accurate than that described in the work of last year.

As before, the octagonal mirror making 528 turns per second rotates through one-eighth of a turn during the time of passage of light from the revolving mirror to the distant station and return, thus presenting the succeeding face of the mirror to the returning beam at (very nearly) the same angle as at rest.

The observations consist, then, in increasing the speed of the revolving mirror until the stroboscopic image between fork and mirror is stationary, at which instant the measurement is taken of the small angle α_1 by which the displacement of the image differs from 90°. The direction of rotation is then reversed, and a similar series of observations furnishes the angle α_2. If $\alpha=\alpha_1+\alpha_2$, then the angle through which the mirror rotates during the time required for light to travel through the distance $2D$ will be $\pi/4-\alpha/4$, and the velocity is given by $V=(16ND)/(1-\alpha/\pi)$.

If $1/n$ is the period of the (optical) beats between the fork and the pendulum and $1/\nu$ that of the coincidences between the C.G.S. pendulum and the true seconds, and if N is the nearest whole number (in the present instance 528), then, as both α and ν are small,

$$V=\frac{16D}{1-\nu}(N+n)\left(1+\frac{\alpha}{\pi}\right),$$

or since α and n are small,

$$V=\frac{16\times 35425.15\times 528}{1-.00051}\left(1+\frac{\alpha}{\pi}+\frac{n}{N}\right) \qquad D=35425.1$$

$$V=299{,}425\left(1+\frac{\alpha}{\pi}+\frac{n}{N}\right).$$

Table I gives the results of ten series of observations with the glass octagon.

These results, as well as those previously published[1] are to be regarded as preliminary. The definitive measurements were begun in June, 1926, and continued until the middle of September.

[1] *Astrophysical Journal*, **60**, 256, 1924.

TABLE I

TEN SERIES OF OBSERVATIONS

	a/π	n/N	V_a
I	0.00077	0.00013	299,695
II	.00057	.00015	299,651
III	.00044	.00038	299,671
IV	.00037	.00047	299,677
V	.00054	.00045	299,722
VI	.00047	.00043	299,695
VII	.00032	.00068	299,725
VIII	.00017	.00070	299,686
IX	.00018	.00076	299,707
X	0.00021	0.00058	299,662
Mean velocity in air			299,689
Correction			+67
V in vacuo			299,756

FIG. 1.—Arrangement of apparatus

The arrangement of apparatus at the home station was slightly different as shown in Figure 1, the advantage being an increase in intensity in consequence of greater effective width of the light beam falling on the revolving mirror at nearly normal incidence, as well as greater symmetry.

With this layout a series of observations with the same small glass octagon gave the results recorded in Table II. The numbers

TABLE II

	a	*b*	*c*	*V*	Wt.
I	0.00059	0.00028	0.00072	299,747	2
II	.00046	.00040	.00073	299,747	2
III	.00045	.00038	.00073	299,738	3
IV	.00057	.00036	.00072	299,762	3
V	.00047	.00033	.00073	299,729	3
VI	.00052	.00038	.00073	299,759	3
VII	.00061	.00041	.00073	299,792	1
VIII	.00048	.00038	.00072	299,744	4
IX	.00049	.00036	.00072	299,741	4
X	.00047	.00040	.00072	299,747	4
XI	.00044	.00042	.00072	299,744	4
XII	0.00042	0.00042	0.00073	299,741	4
Weighted mean				299,746	
Correction				+67	
Velocity *in vacuo*				299,813	

given are the means of three series of observations, each series containing six (double) observations.

On account of the small values of $a=\alpha/\pi$, $b=n/N$, and $c=\nu$, the formula for the velocity may be written

$$V=16DN(1+a+b+c)\,.$$

With $D=35{,}425$ and $N=528$, this gives

$$V=299270+V(a+b+c)\,.$$

The results with the glass octagon are shown in Table II.

Giving these three results the weights 1, 2, and 5, respectively, the weighted mean of all the observations with the glass octagon is $V=299{,}797$.

DEFINITIVE MEASUREMENTS

Toward the close of the work of the summer of 1925 an attempt was made to attain a speed of rotation of 528 turns per second with a glass octagon nearly twice as large as that used in the experiments just described. Owing probably to some defect in the glass, it burst at a speed of 400, thus terminating the work. In order to provide against a repetition of such an accident four mirrors were constructed. Two of these were of glass, of about the same construction, but twice as large, as the small octagon, a photograph of which is given in Figure 2. The first of these had twelve facets and the second sixteen; the diameters being $6\frac{1}{4}$ and $7\frac{1}{2}$ cm, respectively, and the speed of rotations 350 and 264. The flat end of the steel axle on which the glass mirrors are mounted rests on a steel flat with a small aperture for oil, under pressure a trifle less than that which would lift it from its bearing. The driving power is furnished by an air blast issuing through two nozzles and impinging on the vanes of a paddle wheel.

Regulation of the speed was obtained by a counterblast controlled by a valve by means of which the speed could be kept constant for several seconds (as indicated by the immobility of the stroboscopic image) quite sufficient for the measurement of the corresponding small displacement of the image.

The other two mirrors were of nickel steel and furnished by Mr. Elmer A. Sperry. A photograph of the first, an octagon, is shown in Figure 3.

The driving power is furnished by an air blast from four nozzles impinging on the buckets attached to the axle.[1] The flat end of the latter rests on a single ball-bearing.

All four mirrors gave excellent results notwithstanding the fact that time was insufficient for the attainment of very great accuracy in the surfaces. This was particularly noticeable in the steel mirrors, the lack of homogeneity making the process of polishing and figuring much more difficult than in the case of the glass mirrors.[2]

[1] The mirrors actually employed were provided with duplicate bucket wheels and nozzles, allowing of reversal of the direction of rotation.

[2] All are now in process of refiguring in preparation for a resumption of operations during the winter, when atmospheric conditions are likely to be more favorable.

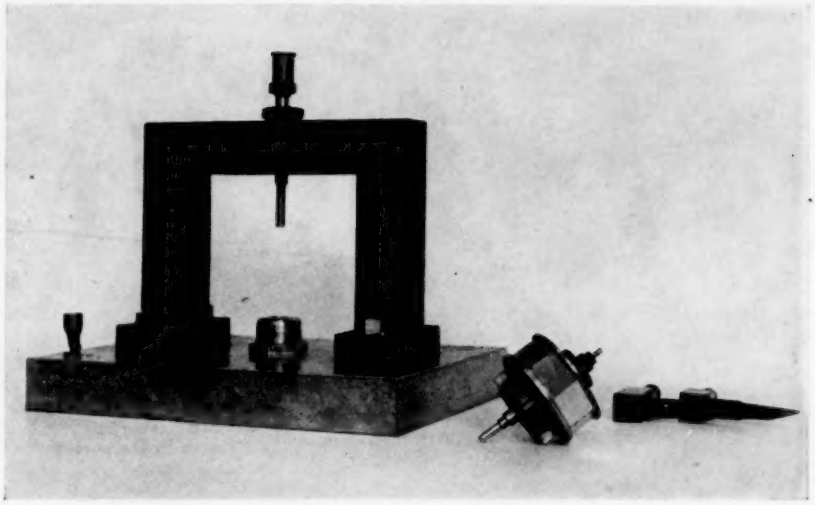

FIG. 2*a*.—The small octagon detached from its mounting

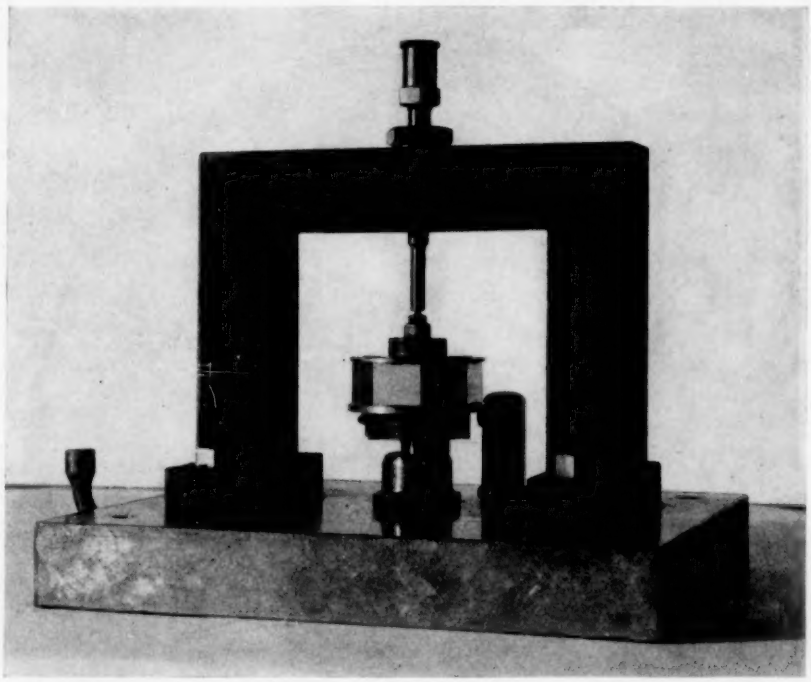

FIG. 2*b*.—The small octagon in its mounting

FIG. 3.—The octagon of nickel steel furnished by Mr. Sperry

With a glass twelve-facet mirror and the layout of apparatus and method of procedure essentially the same as in last year's work, the following results were obtained:

$$V=\frac{24DN}{1-\nu}(N+n)\left(1+\frac{3a}{2\pi}\right)$$

or if

$$\frac{3a}{2\pi}=a\ \frac{n}{N}=b$$

and

$$-\nu=c \qquad D=35424\cdot5$$
$$r=53 \text{ cm}$$
$$V=299{,}265+3(a+b+c)\ .$$

The results obtained are shown in Table III.

TABLE III

TWELVE-FACET GLASS MIRROR

	a	*b*	*c*	*VS*	*V*	Wt.
I	−0.00018	0.00100	0.00075	471	299,736	1
II	.00047	.00040	.00073	480	299,745	3
III	.00058	.00026	.00073	471	299,733	3
IV	.00022	.00061	.00072	465	299,730	3
V	.00012	.00062	.00071	435	299,700	1
VI	− .00007	.00088	.00073	462	299,727	5
VII	.00020	.00098	.00073	453	299,718	5
VIII	.00004	.00085	.00073	462	299,727	5
IX	.00009	.00100	.00073	492	299,757	1
X	− .00021	.00114	.00074	501	299,766	2
XI	.00050	.00037	.00074	483	299,748	2
XII	.00071	.00009	.00073	459	299,724	5
XIII	.00052	.00034	.00073	477	299,742	5
XIV	.00003	.00073	.00075	453	299,718	5
XV	0.00004	0.00071	0.00075	450	299,715	5
Weighted mean					299,729	
Correction					+67	
Velocity *in vacuo*					299,796	

A series of measurements with the glass sixteen-facet mirror the formula for which is

$$V=\frac{32DN}{1+2\ a/\pi} \qquad D=35424\cdot5$$
$$\gamma=55 \text{ cm}$$

gave the results shown in Table IV.

TABLE IV

	a	*b*	*c*	*VS*	*V*	Wt.
I_I	0.00159	−0.00089	0.00076	438	299,703	
I_{II}	.00115	− .00033	.00076	474	299,739	.....
I	$(2\times I_{II}+I_I)/3$	.....	.....	.....	299,727	1
II	.00051	.00045	.00071	501	299,766	2
III	.00038	.00052	.00071	483	299,748	2
IV_I	.00006	.00073	.00071	450	299,715	
IV_{II}	.00079	− .00006	.00073	438	299,703	.....
IV	$(2\times IV_{II}+IV_I)/3$	.....	.....	.....	299,707	5
V	.00090	− .00009	.00073	462	299,727	5
VI_I	.00111	− .00029	.00072	462	299,727	
VI_{II}	.00074	.00013	.00072	477	299,742	.....
VI	$(2\times VI_{II}+VI_I)/3$	.....	.....	.....	299,737	4
VII	.00085	.00011	.00072	504	299,769	3
VIII	.00097	− .00013	.00069	459	299,724	2
IX	.00104	.00008	.00070	498	299,763	2
X	.00081	.00002	.00071	450	299,715	4
XI	.00091	.00007	.00071	465	299,730	4
XII	.00112	.00027	.00069	462	299,727	2
XIII	.00117	.00027	.00069	477	299,742	2
XIV	.00098	.00009	.00070	477	299,742	3
XV	0.00104	−0.00009	0.00070	495	299,760	3
Weighted mean	.....	.....	.....	.....	299,736	.....
Correction	.....	.....	.....	.....	+67	.....
Velocity *in vacuo*	.....	.....	.....	.....	299,803	.....

A second series with the glass sixteen-facet mirror gave the results summarized in Table V.

TABLE V

	a	*b*	*c*	*VS*	*V*	Wt.
I	0.00067	0.00029	0.00071	501	299,766	1
II	.00019	.00060	.00073	456	299,721	5
III	.00020	.00061	.00073	462	299,727	5
IV	.00053	.00029	.00074	468	299,733	1
V	.00039	.00034	.00075	444	299,709	5
VI	.00049	.00029	.00075	459	299,724	5
VII	.00066	.00009	.00073	444	299,709	2
VIII	.00064	.00010	.00073	441	299,706	2
IX	.00078	.00006	.00074	474	299,739	3
X	.00102	− .00015	.00072	477	299,742	3
XI	.00053	.00025	.00073	453	299,718	3
XII	.00059	.00018	.00072	447	299,712	1
XIII	0.00071	0.00023	0.00072	498	299,763	1
Weighted mean	.....	.....	.....	.....	299,722	.....
Correction	.....	.....	.....	.....	+67	.....
Velocity *in vacuo*	.....	.....	.....	.....	299,789	.....

A series of measurements with the steel twelve-facet mirror gave the results shown in Table VI.

TABLE VI

	a	*b*	*c*	*VS*	*V*	Wt.
I_I	0.00063	0.00016	0.00072	453	299,718	.....
I_{II}	.00030	.00046	.00072	444	299,709	
I	$(2\times I_{II}+I_I)/3$	.....	.....	.....	299,712	2
II	.00040	.00043	.00072	465	299,730	4
III	.00051	.00032	.00072	465	299,730	4
IV	.00052	.00030	.00072	462	299,727	5
V	.00052	.00031	.00072	465	299,730	5
VI	.00055	.00031	.00072	474	299,739	5
VII	.00001	.00078	.00072	453	299.718	2
VIII	.00054	.00027	.00073	462	299,727	2
IX	.00056	.000315	.000736	483	299,748	3
X	.00052	.00027	.00074	459	299,724	3
XI	0.00054	0.00023	0.00074	453	299,718	3
Weighted mean	.....	.....	.....	.....	299,729	.....
Correction	.....	.....	.....	.....	+67	.....
Velocity *in vacuo*	.....	.....	.....	.....	299,796	.....

Table VII gives the results obtained with the steel octagon.

TABLE VII

	a	*b*	*c*	*VS*	*V*	Wt.
I	0.00027	0.00057	0.00071	465	299,730	3
II	.00032	.00049	.00071	456	299,721	3
III	.00059	.00028	.00069	468	299,733	3
IV	.00057	.00025	.00069	453	299,718	5
V_I	.00008	.00072	.00072	456	299,712	
V_{II}	.00054	.00030	.00072	468	299,733	.....
V_{III}	.00076	.00005	.00072	459	299,724	
V	Mean of 3	.....	.....	.....	299,723	3
VI_I	.00065	.00027	.00072	492	299,757	3
VI_{II}	.00081	.00005	.00072	474	299,739	.....
VI	$(2\times VI_{II}+VI_I)/3$	.....	.....	.....	299,744	3
VII	.00035	.00049	.00072	468	299,733	3
VIII	.00059	.00024	.00072	465	299,730	5
IX	.00055	.00026	.00072	459	299,724	5
X	.00056	.00025	.00072	459	299,724	5
XI	0.00058	0.00025	0.00072	465	299,730	5
Weighted mean	.....	.....	.....	.....	299,728	.....
Correction	.....	.....	.....	.....	+67	.....
Velocity *in vacuo*	.....	.....	.....	.....	299,795	.....

These results are collected in Table VIII.

TABLE VIII

Mirror	Year	N	n	V	Wt.
Glass 8	1925	528	150	299,802	1
Glass 8	1925	528	200	299,756	1
Glass 8	1926	528	216	299,813	3
Steel 8	1926	528	195	299,795	5
Glass 12	1926	352	270	299,796	3
Steel 12	1926	352	218	299,796	5
Glass 16	1926	264	270	299,803	5
Glass 16	1926	264	234	299,789	5
Weighted mean				299,796±4	

When grouped in series of observations with the five mirrors the results show a much more striking agreement, as follows:

Glass 8	299,797
Steel 8	299,795
Glass 12	299,796
Steel 12	299,796
Glass 16	299,796

The ready success of the measurements at the distance of twenty-two miles, the majority of which were made under conditions not the most favorable (haze and smoke from forest fires; imperfections of surfaces of the revolving mirrors), would seem to indicate the feasibility of a measurement at a considerably greater distance. A convenient location for the distant mirror (diameter, 40 in.) was found at Mount San Jacinto, eighty-two miles distant. In a preliminary trial, light was returned, but so enfeebled by smoke that measurements were quite impracticable.

It is hoped that during the winter season the rains will clear the atmosphere sufficiently for accurate measurement, and accordingly work will begin again toward the end of December, 1926.

I take this opportunity to express my appreciation of the generous gift of $10,000 by Mr. Martin A. Ryerson which made this investigation possible.

APPENDIX I

Measurement of the length $2D$ of the light-path.

The light-path as shown in Figure 1 is from the surface a of the revolving mirror to $b\ c\ D\ E\ f\ E\ D\ c\ b_1\ a_1$.

The measurements of the separate elements are as follows:

Distance between bench marks $B\ B_1$, as furnished by U.S. Coast and Geodetic Survey, 35,385.5

	Meters
$BD=$	11.57
$Dc=$	8.00
$bc=b_1c=$	0.40
$ab=$	0.34
$B_1f=$	0.11
$fE=$	9.30

The D of the formula will be

$$D=ab+bc+cD+DB_1+B_1f+2fE=35,424.5M\ .$$

APPENDIX II (JULY 1)

Comparison between C.G.S. pendulum[1] and standard observatory clock:

Period of Coincidences	Temperature
23^m46^s	24°.0
23 44	24.1
23 24	24.1
23 40	24.2
24 00	24.4
1423	

N = number of (double) swings per second = 1.00070 at 24°.2.

Another comparison gave 1.00074 at 17°.

Whence the temperature correction is $-.0000055$ per degree.

The clock gains one second per day by comparison with time signals from Washington; giving

$$T=1.00075 \text{ at } 17°\ .$$

Another comparison taken August 13 gave the following results:

Period of Coincidences	Temperature
22^m41^s	19°.0
23 15	19.2
23 07	19.4
22 58	19.7
22 50	20.1
23 48	21.0
23 20	21.5

Giving $N=1.00072$ at 19.9°.

[1] The pendulum case was evacuated to a residual pressure of about 1 cm, for which the pressure correction is inappreciable. The amplitude correction was also negligible.

The ratio of the standard clock was $+0^{s}.5$ per day, giving

$$N_2 = 1.000726 \text{ at } 19.9^\circ$$

and

$$N = 1.00074 \text{ at } 17^\circ .$$

APPENDIX III. MEASUREMENT OF LENGTH OF LINE USED IN DETERMINATION OF VELOCITY OF LIGHT

By WILLIAM BOWIE[1]

As a result of several conferences between Professor A. A. Michelson and the writer on the question of the determination, with high accuracy, of the distance between a point on Mount Wilson and one on San Antonio Peak, California, to be used as a base line in the determination of the velocity of light, Professor Michelson made a request on May 1, 1920, upon the director of the United States Coast and Geodetic Survey, Colonel E. Lester Jones, that the work be done. The director, realizing that the determination of the velocity of light with great accuracy might lead to the determination of distance in terms of the velocity of light and thus might furnish a means of measuring base lines in mountainous regions or on archipelagoes, agreed to measure the distance between the two peaks in question.

Professor Michelson requested that the distance be measured with an accuracy of 1 part in 200,000 but with greater accuracy if possible. In a letter to the director, dated March 23, 1922, Professor Michelson made the statement:

> In my conversations and correspondence with Major William Bowie, I was given to understand that the measurement [of the distance between Mt. Wilson and San Antonio Peak] could be made to an order of accuracy of one in a million which would be highly desirable, if possible, but an order of accuracy of one in 100,000 or 200,000 would be sufficient.

In a letter dated March 14, 1922, Professor George E. Hale, director of the Mount Wilson Observatory, requested the director of the United States Coast and Geodetic Survey to determine by triangulation the distance between Mount Wilson and San Antonio Peak for Professor Michelson's use in his redetermination of the ve-

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locity of light. The director of the Coast and Geodetic Survey replied to Dr. Hale, expressing his deep interest in the work on the velocity of light, and made the statement that the field measurements would be undertaken later in the year.

It was realized that to obtain an accuracy in the determination of the length of a line of triangulation as great as 1 in 200,000 or more would necessitate some modifications and refinements in the methods that are usually employed in what is known as first-order triangulation. It is rather difficult to estimate just what is the accuracy of a line of triangulation carried from a base through a series of triangles or quadrilaterals, but it has generally been supposed that the order of accuracy is about 1 part in 100,000.

Ordinarily, the quadrilaterals used in a chain of triangulation approach the square in shape, that being the most satisfactory shape if both accuracy and rapid progress are desired. For the Michelson length it was believed best not to depend on a triangulation extended from the Los Angeles base which had been measured in 1888–1889. There was the possibility that this base had changed its length during the interval since its measurement, and, besides, it was believed that even though the angles of the triangles extending from the Los Angeles base to the Michelson line were remeasured with the greatest care, the lengths carried through the triangles would not have an accuracy of 1 part in 200,000. After careful consideration of the problem, it was decided to measure a base line in the San Gabriel Valley, parallel to the line between Mount Wilson and San Antonio Peak, and to have the base line as close to the foothills as possible in order that the angles involved in transferring the length of the base to the line between the peaks might be of such size as to give the strongest possible connection.

An inspection of the topographic maps of the United States Geological Survey indicated that it was feasible to measure a base of the desired length in this locality. The methods to be employed in the field work were carefully planned at the office of the United States Coast and Geodetic Survey by the members of the Division of Geodesy. The field work was placed in charge of Clem L. Garner, one of the field engineers of the United States Coast and Geodetic Survey who had had a number of years of experience on first-order

triangulation and in the measurement of first-order base lines. In his party were a number of engineers of the Survey who had also had experience in the most accurate geodetic work. They were J. S. Bilby, F. W. Hough, and E. O. Heaton.

The computation of the precise levels on the base line was in charge of H. G. Avers. The computations of the measured lengths were made under the direction of W. D. Sutcliffe. Unusual care was employed to take into account all refinements in computation that could affect the final results. The computation and adjustment of the triangulation to carry the length from the bases to the required line extending from the station Michelson on Mount Wilson to the station Antonio on San Antonio Peak were made under the direction of O. S. Adams. Extra computations were made for the deflection of the vertical, and extreme care was employed at all stages of the work. Other mathematicians engaged upon the work, either directly or in consultation, were W. D. Lambert, C. H. Swick, C. A. Mourhess, Sarah Beall, and W. F. Reynolds.

By a field reconnaissance the ends of the base were located where they could be made intervisible by the erection of observing towers of moderate height and where the two peaks at the ends of the Michelson line were visible from each end of the base. It was found, however, that a straight base from end to end could not be measured, owing to the character of the terrain and to the presence of orange groves and other obstructions. Correspondence between the office and the field resulted in the adoption of a plan to measure a broken base, with the various sections located where the measurements could be most easily made.

The methods adopted for the field measurements and the office computations were such as to assure the attainment of an accuracy, for the straight-line distance between Mount Wilson and San Antonio Peak, corresponding to a probable error of about 1 part in 2,000,000, derived from field measurements and observations alone, and to an actual error surely less than 1 part in 300,000. It is the feeling of those who have been engaged in the work that the actual error is somewhere between 1 part in 500,000 and 1 part in 1,000,000.

It was decided to make the distance between the peaks depend on more than one base. This was accomplished by dividing the total

base into three parts. The length of each part was obtained during the measurement of the whole base line, as the subsidiary stations were placed along the base itself.

The base was measured in forty-six sections, owing to the character of the terrain, to the presence of obstructions, and to the fact that the base cut the property lines at an angle. Each section of the base was measured at least four times, with 50-m. invar tapes which were standardized with extreme accuracy at the Bureau of Standards, Washington, D.C., just before and just after the measurements in the field. The maximum number of measurements of any one section was six. Eight tapes were used in the measurement of the base, and on each section the different measures were made with different tapes.

Along a portion of the line, wooden stakes were set into the ground to support the tapes during the measurements. The length of the tape was transferred by a marker to metal strips nailed to the tops of the stakes at the tape ends. Where the measurement was over a paved highway, special portable tripods made of iron were used. These were set over points which had previously been selected and marked with small iron bolts cemented in drill holes in the pavement. Five supports were used for all the measurements over stakes, and three supports for the measurements over portable tripods and towers. If possible, the intermediate supports were placed in line between the tops of the end supports.

When stakes were used, accurate levels were run over them to determine the difference in elevation of consecutive tape ends. The leveling was done in both directions in order to have a check. Necessarily, where stakes were used the same grade corrections would be applied to each tape length as measured by each of the four or more tapes. On the other hand, when the movable tripods were used the distance of the tripod head above the mark on the road was obtained during the measurement with each tape. Later, when levels were run over these parts of the base to give the differences in elevation of the marked points, these differences were combined with the heights of the tripods to give the data from which to compute the corrections for grade for each tape. Necessarily, the grade corrections were slightly different for the different measurements, owing

to the fact that the tripods could not be set exactly at the same heights for the different measurements.

At places where the line had to be lifted above orange groves, the tapes were supported on towers. Towers were used also to avoid steep grades where measurements were made across rather sharp depressions in the ground. One of the large uncertainties in base measurements, where the ground is very steep, is in the determination of the grade corrections. In order to avoid this uncertainty, the slope for any tape length was seldom allowed to be greater than 10 per cent.

In computing the base, the lengths of the forty-six sections were projected on to straight lines joining the ends of the three main divisions of the base mentioned above. In general, the difference in direction of a section and the line on which it was projected was only a few degrees, and in only one case did it exceed the limit of 14° specified in the instructions, the deflection angle in this instance being 18°.

The uncertainty in the measured length of a base is due to the following causes: errors in the standardization of the tapes, use of erroneous tension on the tapes during the measurements, effect of wind on the tapes, constant and systematic errors in transferring the ends of the tape to the metal strips on the end supports of the tape, errors in leveling from which the grade corrections are computed, uncertainty in the coefficient of expansion of the tape, and errors in the reduction to sea-level from uncertainty in the average elevation of the base line.

The differences in the lengths of the tapes, as derived from the two standardizations at the Bureau of Standards, one before the measurements in the field and the other shortly afterward, are given in Table I.

The effect of the wind on the tape may be considered as having been negligible, for the measurements were made only when there was a very light wind or during spells of calm weather. The use of five-point supports whenever possible also lessened the effect of the wind on the tape.

The errors in transferring the ends of the tape to the metal strips were undoubtedly small, for the observers doing the marking

changed positions between the forward and backward measurements, thus eliminating the systematic effect of any tendency to mark ahead or behind the graduation on the tape. The accidental errors in the marking were very small, owing to the care with which the line on the tape was transferred to the metal strip. The marking was done only when the tape had come to rest after the tension had been applied.

Seven different spring balances were used to apply the tension to the tapes. Each balance was tested twice on every day it was used with a standard 15-kg weight. The balance was placed in a horizontal position during the test, and a small wire led from the tongue of

TABLE I

Tape Nos.	Differences in Length Millimeters
914	+0.17
918	.18
921	.04
925	.03
927	.19
928*	.....
929	.20
933	+0.04

*Owing to a misunderstanding when this tape was sent to the Bureau of Standards, its length was not redetermined. The second standardization gave a shorter length for each of the tapes used.

the balance to the weight over a pulley which was practically frictionless. The balances were also tested at the Bureau of Standards after the field work was completed, and the effect of changes in temperature on the springs of the balances was determined. No attempt was made to set the indexes of the balances to make the dials give correct readings, but in the office computations corrections were applied to the measured lengths to account for the errors of the balances as disclosed by the daily tests and for the effects on the springs of any temperature variations.

Extreme care was used in determining the elevation of the tape supports. Levels were run in both directions over the base and, wherever the two runnings failed to agree within a certain amount, a third running was made as a check. The elevations of the ends of the different sections of the base were determined by first-order leveling, and the elevations, as carried through the differences of

elevation of the tape ends, were checked with the elevations determined by the first-order levels. It may be assumed, therefore, that the corrections for grade had exceedingly small errors.

The temperature range during the measurements of the base was from 5° to 25° C. The tapes were standardized at 26° and 28° C. The coefficient of expansion of each tape had been determined at the Bureau of Standards with such accuracy that it is believed that any error in the length of the base, due to erroneous temperature corrections, is exceedingly small.

The elevation of the base above sea-level was determined by checked first-order leveling from a nearby bench mark of known elevation, so it is reasonably certain that no error was involved in the sea-level correction.

The agreement of the several measurements of each section indicated that the accidental and systematic errors were largely eliminated by the methods employed. As was mentioned earlier, each section was measured at least four times, each time with a different tape. The average residual of a single measure from the mean of all measurements on a section was 1.1 mm. Only three measures were rejected, and in only thirty-two measures of the remaining total of two hundred eight was the residual more than 2 mm. The average size of the probable error of the sections was 0.5 mm, with no probable errors greater than 1 mm. The maximum probable error of a section was 0.9 mm.

The probable error of the length of the base from field measurements and observations alone was 3.45 mm, or 1 part in 11,600,000. The Bureau of Standards certified that the lengths of the tapes were correct within 1 part in 300,000 and that the probable error did not exceed 1 part in 2,000,000.

During the measurement of the base, observations were made for horizontal directions at four points along the base line, including the two ends, and at San Antonio Peak and Mount Wilson. These observations were made with high-grade direction theodolites. The observations were repeated a number of times in order to eliminate, as far as practicable, the effect of the accidental errors, and they were made on each of several nights in order that any systematic errors resulting from atmospheric conditions might be largely eliminated.

The deflection of the vertical was determined at each of the stations in order that corrections could be applied to the observed directions to eliminate the effect of any tilting of the geoidal surface with respect to the spheroidal surface. The mountain masses to the north of the base line evidently deflect the plumb line to a marked degree. The determination of the astronomic latitudes was made at each of the angle stations except at station Joaquin, which is within a mile of Pasadena west base, and at station Michelson, where it had already been determined. It was assumed that the deflection in the meridian at Joaquin would be approximately the same as at Pasadena west base.

The effect of the deflection of the vertical is to tilt the horizontal plate of the theodolite with respect to the spheroid and, therefore, to cause the measured angles to vary somewhat from the true spherical angles. The astronomic longitude had been determined at Mount Wilson some years before. A comparison of it with the geodetic longitude furnished a deflection of the vertical in the prime vertical at that point, thus making it possible to correct the astronomic azimuth at Mount Wilson for the deflection of the vertical. In the adjustment of the triangulation the true azimuth obtained at Mount Wilson was held fixed. This made it possible to determine the approximate deflection of the vertical in the prime vertical at each of the other stations by a comparison of the astronomic azimuth and the geodetic azimuth at each of them.

The results of the computations showed clearly that it was a wise move to determine the deflections at each of the stations, for the maximum correction to a direction was nearly $1''.5$. A number of directions were only slightly affected, but on the whole the application of corrections for the deflections resulted in appreciably greater accuracy in the determination of the length of the Michelson line.

Owing to the fact that the lines from the base stations to the mountain peaks have inclinations as great as $8°$ to the horizontal plane, it was necessary to read the stride level of the theodolite whenever pointings were made over the steep lines, in order that corrections might be applied for any tilting of the horizontal axis of the telescope during the observations. This is a refinement in angle

measurements that has never been applied in the regular triangulation of the country. In fact, the inclination of a line of triangulation is seldom greater than 1° or 2°, and the error from this source is ordinarily negligible.

The heights of the stations on Mount Wilson and on San Antonio Peak were determined from the base stations by trigonometric leveling. These elevations were required for the purpose of determining the air-line distance between the stations on the two peaks.

The average closing error of the triangles involved in the triangulation was 0".55. There were only two triangles with closing errors greater than 1", while the maximum closing error was 1".57.

The average correction to a direction, resulting from the least-square adjustment, was 0".24; and the maximum correction was 0".83.

The distance, reduced to sea-level, between the triangulation stations Michelson on Mount Wilson and Antonio on San Antonio Peak, resulting from the adjustment, is 35373.21 m. The air-line distance between these two stations is 35385.53 m. No correction was applied to the distance to allow for normal curvature of the line of light between the two stations on account of refraction. A computation was made which showed that this correction is only one part in several million.

The probable error of the straight-line distance from Mount Wilson to San Antonio Peak is 1 part in 6,800,000 from field measurements and observations alone. It is believed that the length of this line has been determined with greater accuracy than that of any other line of triangulation in this or any other country.

STELLAR SPECTRA OF CLASS S[1]

II. CHANGES IN SPECTRA OF LONG-PERIOD VARIABLE STARS

By PAUL W. MERRILL

ABSTRACT

Revised list of S-type stars.—A revised list of 31 S-type stars, of which 20 are variable, is given in Table I. The presence of zirconium bands in the spectrum of χ Cygni is noted. The detailed classification of S-type spectra is a problem for the future.

Spectroscopic data.—Spectroscopic observations are listed in Table II, which includes the estimated intensities of the bright lines $H\beta$, $H\gamma$, and $H\delta$, and of the dark zirconium and titanium bands. Several variables, especially S Ursae Majoris and R Cygni, were extensively observed in order to determine the spectral variations during the light-cycle. Detailed descriptions of numerous lines follow Table II.

Characteristic behavior of spectra of variables.—The characteristic behavior of several features in the spectra of long-period variables is summarized. The zirconium bands are slightly weaker near maximum and the titanium bands much weaker. Bright hydrogen lines are absent at minimum and are strongest near maximum. The intensity decreases rapidly from $H\beta$ to $H\delta$ when titanium bands are absent, but $H\beta$ and $H\gamma$ are weaker when these bands are present. Bright enhanced lines of iron are visible at maximum and persist through a considerable portion of the light-cycle. The bright lines $\lambda\lambda$ 4202, 4308, 4511, 4521, 4571, and several others behave about as in Me variables, but certain differences are noted. The dark line λ 4554 *Ba*+ is slightly stronger at maximum than at other phases while λ 4607 *Sr* is much weaker at maximum.

Relationship between classes M and S.—S-type spectra are closely related to those of giant stars of class M, and the temperature ranges of the two types of stars overlap. S-type stars probably have lower densities than M-type stars of the same effective temperatures. This is indicated by the spectra and by the relatively long periods of S-type variables.

Special effects of titanium oxide.—Certain peculiarities of advanced M-type spectra may be special effects of a relatively high stratum of absorbing titanium oxide. S-type stars will be useful in studying this and similar problems of red stars.

A general description of spectra of class S, together with an account of the observations leading to the inclusion of this class in the Draper system of stellar classification, may be found in *Mount Wilson Contribution*, No. 252; *Astrophysical Journal*, **56**, 457, 1922. This description contains references to prominent absorption bands apparently characteristic of class S, whose origin has since been identified with zirconium oxide.[2] It is planned later to publish a detailed discussion of the identification of lines and bands in spectra of this type.

The present contribution deals principally with certain long-period variable stars whose spectra are characterized by the chief

[1] *Contributions from the Mount Wilson Observatory*, No. 325.

[2] *Publications of the Astronomical Society of the Pacific*, **35**, 217, 1923.

features of class S. The spectral changes which accompany the periodic variations in brightness shed a strong light upon the relationship between classes M and S, and have a bearing on many other problems of long-period variables.

A revised list of thirty-one stars of class S is given in Table I. There is some doubt concerning the proper classification of W Cassiopeiae, R Orionis, and R Canis Minoris. These stars have the S-type characteristics, with the important exception of the zirconium oxide bands, which are very weak or absent. A fourth star, not included in Table I, which bears some resemblance to class S but does not exhibit definite zirconium bands, is VX Geminorum,[1] H.D. 55284.

Another question concerning classification is raised by spectra which contain both titanium and zirconium bands. In certain stars, for example π^1 Gruis, the S-type features are dominant and the spectra might be described as class S *plus* titanium bands. In other stars, for example H.D. 22649 (Boss 826), H.D. 35155,[2] and χ Cygni, the S-type features are less dominant, and the spectra appear to be intermediate between classes M and S. Intermediate spectra in which the zirconium and the titanium bands are both comparatively weak are apparently not very numerous, as H.D. 22649 is the only one found among the M-type stars on the absolute magnitude program of the Mount Wilson Observatory.[3]

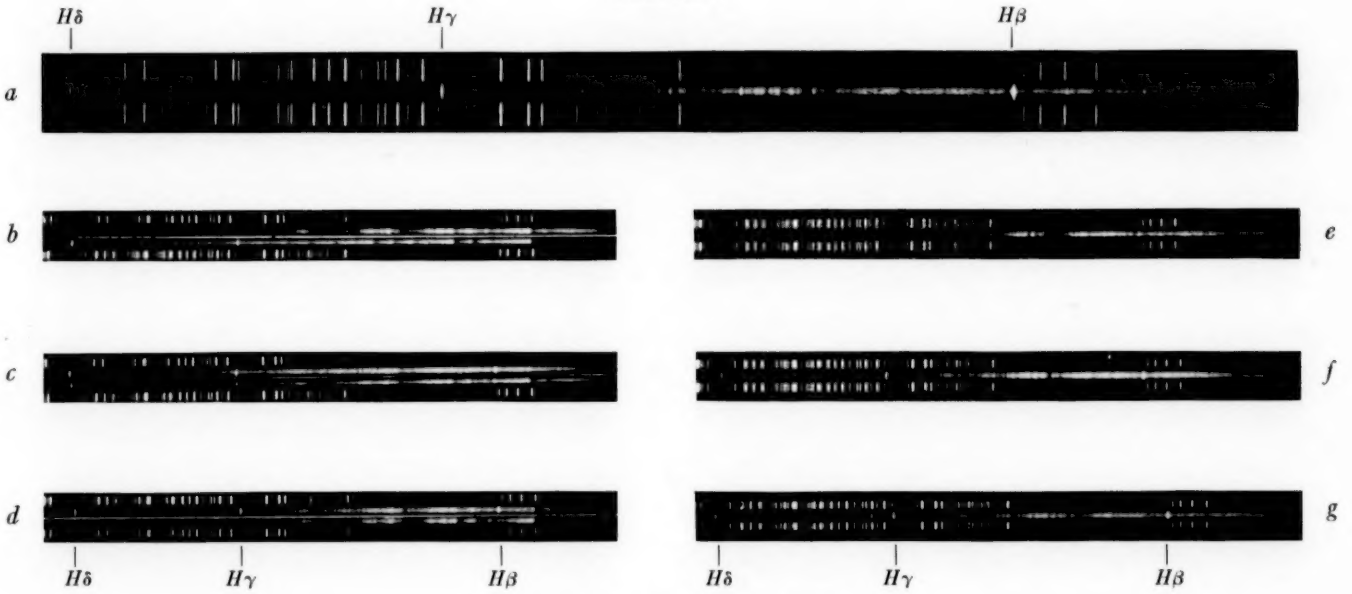
Various combinations of zirconium and titanium bands have been observed in the spectra of variable stars. In a given spectrum both groups of bands may be strong, as in AA Cygni (see Plate I *b*); or the zirconium bands may be strong and the titanium weak, as in R Andromedae, or absent, as in S Ursae Majoris. In several variables the titanium bands undergo large variations in intensity during the light-cycle. In one well-known long-period variable, χ Cygni, zirconium bands of moderate intensity are associated with very strong titanium bands. In this case, the M-type spectrum appears dominant and the star is not included in Table I. It may,

[1] See remark in the *Henry Draper Catalogue;* also Sanford, *Mt. Wilson Contr.*, No. 276; *Astrophysical Journal*, **59**, 339, 1924.

[2] See Plate XII in *Mt. Wilson Contr.*, No. 252; *Astrophysical Journal*, **56**, 457, 1922.

[3] *Mt. Wilson Contr.*, No. 319; *Astrophysical Journal*, **64**, 225, 1926.

PLATE I



SPECTRA OF S-TYPE VARIABLE STARS

a) R Cygni, γ 11117; mag. 8.0; phase +32 days.
b) Upper, AA Cygni, C3377; lower, RS Ursae Majoris, C3241, April 9, 1925; spectral class M5e.
c) U Cassiopeiae: upper, C3074; mag. 7.4; phase −11 days: lower, C3538; mag. 9.0; phase +22 days.
d) T Geminorum: upper, C3240; mag. 8.9; phase −10 days: lower, C 3186; mag. 9.4; phase −47 days.
e) R Cygni, C3492; mag. 9.6; phase −61 days.
f) R Cygni, C3572; mag. 6.8; phase − 3 days.
g) R Cygni, C1055; mag. 9.7; phase +115 days.

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however, eventually prove expedient to classify such spectra with those of class S.

It will doubtless become desirable to form subdivisions of class S, analogous to those of class M, depending on the intensity of the

TABLE I

STARS OF CLASS S

Name	H.D.	R.A. 1900	Dec. 1900	Mag.	Spect.	Period, Days
X Androm.	1167	$0^h10^m.9$	+46° 27′	8.1–14.2	Se	346.5
R Androm.	1967	0 18.8	+38 1	5.6–14.0	Se	410.7
U Cassiop.	4350	0 40.8	+47 43	8.0–16	Se	276.0
W Cassiop.	5235	0 49.0	+58 1	8.4–12.2	(Se)	403.5
S Cassiop.	7769	1 12.3	+72 5	7.6–14.5	Se	609.5
Boss 826.	22649	3 33.5	+62 54	5.3	S	.....
T Camelop.	29147	4 30.3	+65 57	7.0–13.5	Se	370
R Orionis.	31798	4 53.6	+ 7 59	8.7–13.5	(Se)	378.5
	35155	5 17.6	− 8 45	7.0	S	.....
R Lyncis.	51610	6 53.0	+55 27	7.0–13.8	Se	379.2
R Gemin.	53791	7 1.3	+22 52	6.4–13.8	Se	370
R Can. Min.	54300	7 3.2	+10 11	7.2–10.0	(Se)	337.7
	58881	7 22.4	−11 31	9.0	S	.....
	62164	7 37.5	−10 39	Var.	S	.....
T Gemin.	63334	7 43.3	+23 59	8.0–13.5	Se	288.1
	63733	7 45.3	−18 45	8.5	S	.....
	64332	7 48.3	−11 22	8 1	S	.....
V Cancri.	70276	8 16.0	+17 36	7.5–13.0	Se	272.1
S Urs. Maj.	110813	12 39.6	+61 38	7.3–12.5	Se	226.5
	121447	13 50.3	−17 45	8.1	S	.....
R Camelop.	127226	14 25.1	+84 17	7.9–13.7	Se	269.5
RW Librae.	136734	15 17.2	−23 42	8.6–11.7	(Se)	490 (about)
	156957	17 15.3	−41 39	8.4	S	.....
	172804	18 37.1	+ 6 43	9.1	S	.....
	177175	18 58.5	+12 7	8.7	S	.....
T Sagittarii.	180196	19 10.5	−17 19	7.2–<13.1	Se	381.3
R Cygni.	185456	19 34.1	+49 59	6.6–13.9	Se	425.9
AA Cygni.	190629	20 0.8	+36 32	8.4– 9.2	S	Unknown
S Cygni.	.....	20 3.4	+57 42	9.2–<14.7	Se	323
Z Delphini.	195763	20 28.1	+17 7	8.2–14.0	Se	303.4
π^1 Gruis.	212087	22 16.6	−46 27	6.6	S	.....

characteristic features, especially of the zirconium bands, and to provide a nomenclature for spectra intermediate between classes M and S; but the detailed discussion of these matters should perhaps await further experience.

The journal of observations and certain data concerning the individual spectrograms are given in Table II. The meaning of the letters preceding the plate numbers in the first column is as follows:

C—One-prism spectrograph, attached at the Cassegrain focus of the 100-inch Hooker telescope. Two camera lenses were employed with focal lengths of 10 and 18 inches, and dispersions at $H\gamma$ of 75 and 40 A per mm, respectively.

γ—One-prism spectrograph, attached at the Cassegrain focus of the 60-inch telescope. Nearly all the plates were taken with the 18-inch camera, having a dispersion at $H\gamma$ of 34 A per mm.

H—Small ultra-violet spectrograph, attached at the Newtonian focus of the 100-inch telescope. Dispersion at $H\gamma$, 95 A per mm.

G—Plane-grating spectrograph for the yellow and red regions, attached at the Cassegrain focus of the 100-inch telescope. Dispersion at 66 A per mm.

R—Three-prism spectrograph for the yellow and red regions, attached at the Cassegrain focus of the 100-inch telescope. Average dispersion from λ 5900 to λ 6700, 40 A per mm.

The source for plate numbers without prefix is indicated in the notes following the table.

The column headed "Phase" gives the number of days before (−) or after (+) the nearest maximum. Most of the photometric data were supplied by the Harvard College Observatory through the courtesy of Dr. Shapley and Mr. Campbell. I wish to thank these gentlemen for their continued co-operation in furnishing data concerning variable stars.

The intensities of the bright hydrogen lines are estimates of the strength of the lines on the negatives, no allowance having been made for instrumental or other observational effects. The $H\beta$ line is often overexposed, and in these cases (say intensity greater than 10) the estimates of this line are doubtless only rough approximations. The estimated intensities of the absorption bands of zirconium oxide and titanium oxide are given next. The figures for the titanium bands denote the subdivision of class M to which the intensity of the bands corresponds. Subdivisions of class S have not yet been formed, but the estimates of the zirconium bands have been made on somewhat the same basis. These estimates may be interpreted as follows: 0, not seen; 1, weak; 2, medium intensity; 3, strong; 4, very strong; "pr," present; "tr," trace. Parentheses indicate uncertainty.

Notes in the last column indicate the portion of the spectrum shown by each spectrogram, if it is other than the usual blue-violet region. Remarks concerning the effective exposure refer to the region from λ 4600 to λ 4800, unless another portion of the spectrum is mentioned.

TABLE II

JOURNAL OF OBSERVATIONS

PLATE	DATE	J.D. 242	VIS. MAG.	PHASE	INTENSITY EMISSION LINES			INTENSITY ABSORPTION BANDS		CONTINUOUS SPECTRUM
					Hβ	*Hγ*	*Hδ*	ZrO_2	TiO_2	
X Andromedae										
C 1454..	1921 Nov. 13	3007	9.5	− 16	12	2	....	2	0	
C 1462..	Nov. 14	3008	9.5	− 15	4	0.5	....	pr	0	Trace
C 1943..	1922 Oct. 8	3336	9.3	− 24	4	....	....	pr!	0	Green to red
C 2408..	1923 Aug. 29	3661	11.1	− 45	4	....	....	(2)	0	Underexp.
C 2468..	Sept. 30	3693	9.8	− 13	3	....	....	....	....	Not seen
γ 12176.	Oct. 3	3696	9.7	− 10	8	0.5	....	(2)	0	Underexp.
C 2511..	Oct. 26	3719	9.5	+ 13	20	6	1	2	0	
C 2566..	Nov. 28	3752	10.2	+ 46	20	5	tr	(3)	0	
C 2888..	1924 July 18	3985	11.8	− 55	2	....	....	tr	0	Underexp.
C 3015..	Sept. 15	4044	8.8	+ 4	30	10	2	2	0	
R Andromedae										
3238..	1915 May 30	0648	7.3	+ 5	1.5	tr	tr	pr	....	Trace
3291..	July 5	0684	8.5	+ 41	1.5	tr	tr	pr	....	Trace
γ 8725..	1919 Oct. 3	2235	7.5	− 25	5	2	1	3	1	
C 130...	Oct. 15	2247	7.5	− 13	4	2	2	2	1	
C 137...	Oct. 16	2248	7.5	− 12	6	3	3	2	1	
C 1499..	1921 Dec. 15	3039	7.5	− 16	12	5	5	2	(0)	Violet to yellow; overexp.
C 3389..	1925 July 7	4339	9.0	+ 39	7	2	2	(4)	(3)	Underexp.
C 3495..	Sept. 3	4397	10.5	+ 97	3	0.5	0.5	(3)	(4)	Underexp.
U Cassiopeiae										
C 1338..	1921 Sept. 21	2954	10.8	− 56	....	....	....	....	(4)	Trace
C 1449..	Nov. 12	3006	8.2	− 4	8	4	4	2	2	
C 1455..	Nov. 13	3007	8.2	− 3	9	4	3	2	2	
C 1873..	1922 Sept. 8	3306	9.3	+ 19	3	2	4	3	5	
C 2363..	1923 July 27	3628	11.5	+ 67	1	0.5	....	....	(4)	Trace
C 3074..	1924 Nov. 10	4100	7.4	− 11	12	7	6	2	2	
C 3096..	Dec. 13	4133	8.1	+ 22	12	9	6	(2)	2	Overexp.
C 3113..	1925 Jan. 6	4157	9.2	+ 46	10	9	6	3	3	Overexp.
C 3494..	Sept. 2	4396	8.9	− 6	2	1	3	3	4	
C 3538..	Sept. 30	4424	9.0	+ 22	3	2	4	4	5	
W Cassiopeiae										
C 3918..	1926 July 26	4723	8.2	− 18	8	4	....	(0)	0	
C 3927..	Aug. 10	4738	8.0	− 3	12	7	1	0	0	
S Cassiopeiae										
γ 10755.	1922 Jan. 13	3068	8.9	+ 22	6	2	tr	....	....	Trace
γ 11985.	1923 July 29	3630	7.7	− 1	12	4	1	2	1	
γ 12100.	Sept. 2	3665	8.4	+ 34	10	3	0.5	2	2	

TABLE II—*Continued*

Plate	Date	J.D. 242	Vis. Mag.	Phase	Intensity Emission Lines			Intensity Absorption Bands		Continuous Spectrum
					$H\beta$	$H\gamma$	$H\delta$	ZrO_2	TiO_2	
H.D. 22649										
γ 7471..	1918 Oct. 20	1887	5.3	.....	....	....	....	1	3	
γ 7838..	1919 Jan. 22	1981	.....	.....	....	....	....	1	3	
γ 8896..	Dec. 8	2301	.....	.....	....	....	....	1	3	
C 1934..	1922 Oct. 7	3335	.....	.....	....	....	....	pr	3	Blue to red
C 2071..	1923 Jan. 8	3428	.....	.....	....	....	....	pr	pr	Green to red
T Camelopardalis										
γ 11245.	1922 Aug. 9	3276	9.0	+ 59	5	2	....	3	1	Underexp.
R Orionis										
C 2409..	1923 Aug. 29	3661	9.3	+ 11	10	5	0.5	0	0	
C 2424..	Sept. 1	3664	9.3	+ 14	1	0.4	....	0	0	Red
C 3017..	1924 Sept. 15	4044	10.0	− 10	10	5	0.5	0	0	
H.D. 35155										
C 672...	1920 Sept. 26	2594	7.0	.....	....	....	....	1	3	
C 2065..	1923 Jan. 7	3427	.....	.....	....	....	....	pr	pr	Green to red
R Lyncis										
3122..	1915 Feb. 28	0557	8.2	− 4	8	3	1	2	0	
3123..	Feb. 28	0557	8.2	− 4	7	3.5	1	2	0	
3140..	Mar. 12	0569	8.1	+ 8	5.5	3	0.5	2	0	
3160..	Mar. 28	0585	8.3	+ 24	3	2	....	....	....	Not seen
C 2512..	1923 Oct. 26	3719	11.3	+124	3	0.5	....	(3)	0	
C 3539..	1925 Sept. 30	4424	9.9	+ 62	10	4	1	3	1	
R Geminorum										
γ 8727..	1919 Oct. 3	2235	7.4	− 27	8	3	0.5	2	0	
C 127...	Oct. 14	2246	7.0	− 16	10	5	2	2	0	
C 133...	Oct. 15	2247	7.0	− 15	12	6	2	2	0	
C 144...	Oct. 17	2249	7.0	− 13	12	6	2	2	0	
C 238...	1920 Jan. 9	2333	8.7	+ 75	0.5	tr	....	....	....	Not seen
C 681...	Sept. 28	2596	7.2	− 30	8	3	....	3	0	
C 821...	Dec. 27	2686	8.3	+ 60	10	4	0.5	3	0	
C 1339..	1921 Sept. 21	2954	8.1	− 50	4	0.5	....	2	0	
γ 10524.	Oct. 10	2973	7.2	− 31	12	5	1	2	0	
C 1402..	Oct. 13	2976	7.1	− 28	....	....	....	pr	0	Green to red
C 1408..	Oct. 14	2977	7.1	− 27	....	....	....	pr	0	Green to red
C 1456..	Nov. 13	3007	7.2	+ 3	15	7	2	2	0	
C 1501..	Dec. 15	3039	7.3	+ 35	12	7	2	3	0	
C 1502..	Dec. 15	3039	7.3	+ 35	7	3	tr	2	0	Blue to yellow
C 2003..	1922 Nov. 6	3365	6.6	+ 10	25	15	7	(2)	(0)	Overexp.

TABLE II—*Continued*

Plate	Date	J.D. 242	Vis. Mag.	Phase	Intensity Emission Lines			Intensity Absorption Bands		Continuous Spectrum
					Hβ	*Hγ*	*Hδ*	ZrO_2	TiO_2	
R Geminorum—*Continued*										
C 2062..	1923 Jan. 6	3426	7.6	+ 71	10	6	1.5	3	o	
C 2480..	Oct. 3	3696	9.1	− 58	3	1	....	3	o	
C 2515..	Oct. 27	3720	7.8	− 34	5	2	....	3	o	
C 2519..	Oct. 28	3721	7.8	− 33	6	1	....	3	o	
C 2563..	Nov. 27	3751	7.3	− 3	8	2	....	3	o	
C 2701..	1924 Feb. 21	3837	9.2	+ 83	12	4	tr	3	o	
γ 13098.	Nov. 11	4101	6.8	− 9	16	8	2	2	o	
C 3097..	Dec. 13	4133	7.0	+ 23	25	12	6	2	o	Overexp.
C 3574..	1925 Oct. 30	4454	6.8	− 27	20	10	2	2	o	
H 57....	Nov. 16	4471	6.8	− 10	15	7	2	pr	(o)	
H 61....	Nov. 17	4472	6.8	− 9	30	16	5	pr	(o)	Overexp.
C 3623..	Dec. 26	4511	7.1	+ 30	24	11	5	2	o	
C 3647..	1926 Jan. 4	4520	7.2	+ 39	14	7	3	3	o	Violet to red
C 3671..	Jan. 25	4541	7.7	+ 60	16	8	4	2	o	
C 3694..	Feb. 21	4568	9.0	+ 87	15	7	3	2	o	
R Canis Minoris										
C 294...	1920 Mar. 3	2387	8.0	− 10	15	5	0.5	o	o	
C 299...	Mar. 4	2388	8.0	− 9	15	5	0.5	o	o	
γ 9047..	Mar. 7	2391	8.0	− 6	8	3	....	....	o	Trace
C 909...	1921 Feb. 25	2746	8.3	+ 20	15	7	2	o	o	
γ 9990..	Feb. 28	2749	8.3	+ 23	....	....	....	....	....	Red; trace
C 958...	Mar. 26	2775	8.4	+ 49	1	0.5	....	....	....	Not seen
C 1522..	1922 Jan. 12	3067	8.5	− 33	6	3	....	....	....	Underexp.
C 2015..	Dec. 8	3397	8.1	− 42	12	5	0.5	o	o	
C 2067..	1923 Jan. 7	3427	8.1	− 12	4	....	....	o	o	Yellow to red
C 2157..	Mar. 8	3487	8.4	+ 48	1	tr	....	....	....	Not seen
H 5.....	Mar. 23	3502	8.9	+ 63	1	0.5	....	?	o	
C 2520..	Oct. 28	3721	9.4	− 77	0.5	....	....	....	....	Not seen
C 2568..	Nov. 28	3752	8.3	− 46	tr	....	....	o	o	Red
C 3573..	1925 Oct. 30	4454	7.6	− 4	14	7	1	o	o	
H 58....	Nov. 16	4471	7.6	+ 13	14	6	1	?	o	
C 3615..	Dec. 24	4509	8.2	+ 51	2	1	....	o	o	Yellow to red
H.D. 58881										
C 2191..	1923 Apr. 3	3513	9.0	.....	....	....	....	2	o	
H.D. 62164										
C 3175..	1925 Feb. 11	4193	(8.5)	.....	....	....	....	2	o	
T Geminorum										
C 968...	1921 Mar. 28	2777	9.1	− 23	2	0.5	0.5	(3)	3	Underexp.
C 1009..	Apr. 29	2809	8.6	+ 9	10	5	3	2	1	
C 2072..	1923 Jan. 8	3428	9.9	+ 65	1	0.5	....	3	3	
C 2516..	Oct. 27	3720	9.9	+ 60	3	2	1	3	5	

TABLE II—*Continued*

Plate	Date	J.D. 242	Vis. Mag.	Phase	Intensity Emission Lines			Intensity Absorption Bands		Continuous Spectrum
					$H\beta$	$H\gamma$	$H\delta$	ZrO_2	TiO_2	
T Geminorum—*Continued*										
C 2747..	1924 Apr. 17	3893	10.6	− 68	....	....	tr	pr	(4)	Trace
C 3171..	1925 Feb. 10	4192	10.2	− 68	....	....	....	3	6	
C 3186..	Mar. 3	4213	9.4	− 47	0.5	tr	2	3	6	
C 3240..	Apr. 9	4250	8.9	− 10	7	4	3	2	3	
C 3279..	May 5	4276	8.9	+ 16	6	3	2	2	3	
C 3620..	Dec. 25	4510	9.0	− 40	1	0.5	1	pr!	(3)	Blue to red
C 3672..	1926 Jan. 25	4541	8.7	− 9	16	9	6	2	2	Overexp.
γ 14019.	Jan. 28	4544	8.7	− 6	7	4	2	2	2	
γ 14024.	Feb. 18	4565	8.9	+ 15	13	7	4	2	2	
H.D. 63733										
C 1464..	1921 Nov. 14	3008	8.5	.....	....	....	....	1	2	
H.D. 64332										
γ 9847..	1920 Dec. 29	2688	8.1	.....	....	....	....	2	4	
V Cancri										
C 194...	1919 Nov. 9	2272	8.0	+ 10	15	7	2	tr	o	
γ 8875..	Nov. 13	2276	8.0	+ 14	7	3	1	tr	o	
γ 8883..	Nov. 14	2277	8.0	+ 15	6	3	0.7	tr	o	Underexp.
γ 10132.	1921 Apr. 27	2807	7.9	− 3	10	5	0.7	tr	o	
γ 10757.	1922 Jan. 13	3068	7.9	− 12	9	5	0.7	tr	o	
C 1552..	Feb. 11	3097	7.8	+ 17	15	8	3	tr	o	
C 2743..	1924 Apr. 16	3892	8.0	+ 15	18	9	2	tr	o	
C 3117..	1925 Jan. 6	4157	8.1	− 14	5	3	tr	pr	o	Blue to red
C 3673..	1926 Jan. 25	4541	12.0	+ 99	a	?	a	o	o	
S Ursae Majoris										
3193..	1915 Apr. 25	0613	8.5	− 55	tr	....	....	....	....	Not seen
3200..	May 7	0625	8.5	− 43	tr	....	....	....	....	Not seen
3254..	June 11	0660	8.1	− 8	2	2−	....	pr	(o)	Underexp.
3271..	June 25	0674	7.8	+ 6	3	4	....	pr	(o)	Underexp.
C 369...	1920 Apr. 8	2423	8.5	− 31	7	3	....	2	o	
C 373...	Apr. 10	2425	8.5	− 29	11	5	....	2	o	
γ 9252..	June 3	2479	8.2	+ 25	6	3	....	2	o	
γ 9848..	Dec. 29	2688	8.4	+ 18	6	4	1	2	o	
C 866...	1921 Jan. 28	2718	8.7	+ 48	4	2	....	3	o	
C 926...	Feb. 27	2748	10.0	+ 78	tr	....	....	(3)	o	Underexp.
C 1169..	July 23	2894	8.1	− 3	9	4	1	1	o	
C 1553..	1922 Feb. 11	3097	8.5	− 43	7	3	....	2	o	
C 1626..	Mar. 19	3133	7.6	− 7	9	6	1	1	o	
C 2068..	1923 Jan. 7	3427	10.3	+ 65	....	....	....	pr!	o	Red
γ 11744.	May 1	3541	8.5	− 42	10	4	....	(2)	o	
C 2286..	June 4	3575	8.0	− 8	11	5	1	1	o	

TABLE II—*Continued*

Plate	Date	J.D. 242	Vis. Mag.	Phase	Intensity Emission Lines			Intensity Absorption Bands		Continuous Spectrum
					Hβ	*Hγ*	*Hδ*	ZrO_2	TiO_2	
S Ursae Majoris—*Continued*										
C 2340..	July 3	3604	8.1	+ 21	12	6	2	1	0	
C 2648..	1924 Jan. 22	3807	8.0	− 19	12	6	1	2	0	
C 2702..	Feb. 21	3837	8.2	+ 11	10	5	1	2	0	
C 2748..	Apr. 17	3893	10.0	+ 67	tr	....	....	3	0	
C 2816..	June 12	3949	11.0	− 94	....	....	....	pr	0	
C 3118..	1925 Jan. 6	4157	11.2	+114	....	....	....	pr	....	Red; underexp.
C 3172..	Feb. 11	4193	10.3	− 73	....	....	....	(3)	0	
C 3187..	Mar. 3	4213	8.5	− 53	9	4	....	3	0	
H.D. 121447										
C 1706..	1922 May 16	3191	8.1	.....	....	....	....	1	0	
γ 11842.	1923 June 5	3576	.....	.....	....	....	....	....	0	Blue to yellow; poor
C 2745..	1924 Apr. 16	3892	.....	.....	....	....	....	0	0	
C 2823..	June 14	3951.	.....	.....	....	....	....	1	0	
R Camelopardalis										
γ 8266..	1919 June 9	2119	8.2	+ 7	4	2	....	....	....	Not seen
γ 8272..	June 10	2120	8.2	+ 8	2−	1	....	....	....	Not seen
γ 8345..	July 8	2148	8.9	+ 36	2	1	....	pr	(2)	Trace
γ 11077.	1922 May 17	3192	8.2	− 18	4	2	0.4	pr	....	Trace
γ 11603.	1923 Feb. 5	3456	8.6	− 23	2	1	....	2	2	Underexp.
γ 11702.	Apr. 7	3517	8.6	+ 38	2	1	tr	2	3	Underexp.
RW Librae										
C 2649..	1924 Jan. 22	3807	(8.5)	.....	12	8	3	?	1	
C 3188..	1925 Mar. 3	4213	(9.5)	.....	tr	4	2	?	5	
C 3242..	Apr. 9	4250	(9.3)	.....	1	4	3	0	5	
C 3281..	May 5	4276	(10.0)	.....	....	tr	....	....	pr	Trace
C 3375..	July 5	4337	(10.0)	.....	....	....	....	....	....	Trace
C 3780..	1926 Apr. 30	4636	(8.5)	.....	4	3	1	0	2	
H.D. 156957										
C 1130..	1921 July 14	2885	8.4	.....	....	....	....	(3)	4	
C 2878..	1924 July 16	3983	.....	.....	....	....	....	3	4	
H.D. 172804										
C 472...	1920 June 6	2482	9.1	.....	....	....	....	3	1	
H.D. 177175										
C 3491..	1925 Sept. 2	4396	8.7	.....	....	....	....	2	5	

TABLE II—*Continued*

Plate	Date	J.D. 242	Vis. Mag.	Phase	Intensity Emission Lines			Intensity Absorption Bands		Continuous Spectrum
					$H\beta$	$H\gamma$	$H\delta$	ZrO_2	TiO_2	
T Sagittarii										
C 1059..	1921 June 20	2861	(8.5)	+ 39	7	3	1	pr	....	Trace
C 1792..	1922 July 13	3249	9.1	+ 65	6	2	....	pr	(0)	Trace
C 2365..	1923 July 28	3629	8.9	+ 47	4	1	tr	pr	....	Trace
C 2410..	Aug. 30	3662	9.8	+ 80	5	1	....	(2)	0	Underexp.
C 2817..	1924 June 12	3949	9.0	− 41	2	....	....	4	1	
C 2887..	July 18	3985	8.1	− 5	20	8	3	3	1	
R Cygni										
345...	1911 Dec. 5	9376	6.7	+ 5	8	3	tr	(3)	0	
349...	Dec. 11	9382	6.8	+ 11	pr	....	....	pr!	0	Blue to red
	1912 Jan. 13	9415	7.8	+ 44	8	3	2	....	....	
C 961...	1921 Mar. 26	2775	7.2	+ 30	15	6	3	3	0	
C 965...	Mar. 27	2776	7.3	+ 31	15	6	2	3	0	
C 966...	Mar. 27	2776	7.3	+ 31	....	....	....	pr	....	Red; underexp.
C 1007..	Apr. 28	2808	8.9	+ 63	15	5	2	4	1	
C 1038..	May 26	2836	9.2	+ 91	15	5	2	4	0	
C 1055..	June 19	2860	9.7	+115	20	7	2	3	0	
C 1171..	July 23	2894	10.6	+149	7	2	tr	3	0	
γ 11079.	1922 May 17	3192	6.8	+ 4	10	4	1	4	0	
γ 11117.	June 14	3220	8.0	+ 32	15	5	2	4	0	
C 1784..	July 11	3247	8.9	+ 59	15	6	2	4	1	
C 1948..	Oct. 9	3337	11.5	+149	....	....	....	pr!	0	Red
γ 11746.	1923 May 1	3541	8.8	− 47	4	....	....	(3)	0	
C 2245..	May 5	3545	8.5	− 43	9	1	tr	4	0	
R 5.....	June 2	3573	6.5	− 15	....	....	....	pr!	0	Red
γ 11843.	June 5	3576	6.4	− 12	18	10	5	3	0	
γ 11912.	June 30	3601	6.5	+ 13	20	10	6	3	0	
R 12....	July 1	3602	6.6	+ 14	....	....	....	pr!	0	Red
R 14....	July 2	3603	6.6	+ 15	....	....	....	pr!	0	Yellow to red
C 2411..	Aug. 30	3662	8.5	+ 74	20	10	4	3	0	
C 2474..	Oct. 2	3695	9.8	+107	10	4	1	3	0	
C 2510..	Oct. 26	3719	10.3	+131	7	2	tr	3	0	
C 2820..	1924 June 13	3950	11.9	− 80	1	....	....	(4)	0	
C 2886..	July 18	3985	9.5	− 45	10	3	tr	3	0	
C 2918..	Aug. 9	4007	7.8	− 23	9	3	0.5	3	0	
C 3019..	Sept. 16	4045	8.2	+ 15	15	6	2	3	0	
G 25....	Oct. 4	4063	8.5	+ 33	....	....	....	pr!	0	Red
C 3492..	1925 Sept. 2	4396	9.6	− 61	5	tr	....	4	0	
C 3536..	Sept. 30	4424	7.4	− 33	12	4	2	3	0	
C 3572..	Oct. 30	4454	6.8	− 3	15	6	2	3	0	
H 59....	Nov. 17	4472	6.9	+ 15	16	8	3	pr	(0)	
AA Cygni										
C 3013..	1924 Sept. 15	4044	(8.5)	.....	....	....	....	3	4	
C 3377..	1925 July 5	4337	(9.0)	.....	....	....	....	4	5	

TABLE II—*Continued*

Plate	Date	J.D. 242	Vis. Mag.	Phase	Intensity Emission Lines			Intensity Absorption Bands		Continuous Spectrum
					Hβ	*Hγ*	*Hδ*	ZrO_2	TiO_2	
S Cygni										
C 458...	1920 June 2	2478	8.9	− 2	2	2	1	....	....	Trace
C 3926..	1926 Aug. 10	4738	10.0	− 1	6	5	3	1	3	
Z Delphini										
C 2565..	1923 Nov. 28	3752	8.6	− 12	11	9	7	1	3	
C 3493..	1925 Sept. 2	4396	8.9	+ 21	10	9	6	1	3	
C 3537..	Sept. 30	4424	10.4	+ 49	4	5	3	tr?	5	
π^1 Gruis										
6425 I...	1915 July 17	0696	6.6	.....	....	....	....	4	3	

NOTES ON INDIVIDUAL STARS

X Andromedae

Hα is a strong emission line on Plate C 1943.

R Andromedae

The first two plates were taken with the 37½-inch reflector at Ann Arbor.[1] *Hζ* is present as a bright line on Plates C 137 and C 1499. *Hε* is not seen on any of the plates.

The bright lines λ 4308 and λ 4571 are present on Plate C 3495.

The zirconium bands, at least certain features of them, are stronger than in R Geminorum. The titanium bands are faintly present near maximum and develop to considerable strength as the light decreases. In R Geminorum, on the other hand, they do not appear on any of my spectrograms. The enhanced barium absorption line λ 4554 is weaker than in the spectrum of R Geminorum. These facts probably indicate a cooler reversing layer than that of R Geminorum.

Plate C 137 is reproduced in Plate XII *a*, *Mt. Wilson Contr.*, No. 252; *Astrophysical Journal*, **56**, 457, 1922.

U Cassiopeiae

Bright *Hζ* is present on nearly all of the plates, but *Hε* is seen on C 3113 only. *Hε* evidently increased considerably in relative intensity in the interval from December 13, 1924, to January 6, 1925, because *Hζ* is a well-marked line on both dates, but *Hε* is not certainly seen on Plate C 3096, taken on the first date, whereas it is approximately as intense as *Hζ* on C 3113 taken on the second date.[2] Several other features increased in

[1] *Publications of the Astronomical Observatory, University of Michigan*, **2**, 45, 1916.

[2] The increase in the relative intensity of *Hε* after maximum light has been observed in Me variables also. See *Mt. Wilson Contr.*, No. 200; *Astrophysical Journal*, **53**, 185, 1921; also Mr. Joy's observations of *o* Ceti, *Mt. Wilson Contr.*, No. 311; *Astrophysical Journal*, **63**, 281, 1926. The phenomenon is probably connected with the decrease in effective absorption of the line H of calcium.

intensity during this interval of twenty-four days, notably the titanium bands and a bright line at λ 4215.

The following bright lines, in addition to those of hydrogen, were measured on Plate C 3113: λλ 4138, 4178, 4202, 4215, 4308, 4521. There is a trace of a bright line at λ 4571. The absorption lines on this plate show little contrast with the continuous spectrum. Weak bright lines are seen at λ 4202 and λ 4308 on Plate C 3096.

Plate I *c* illustrates the change in intensity of the titanium bands.

W Cassiopeiae

Remark in *H.D.*: "The spectrum is peculiar in the distribution of its light. Nearly all of the brightness is between $H\beta$ and $H\gamma$." Remark by Miss Cannon in *Harvard College Observatory Circular*, No. 221: "The continuous spectrum is faint, but it is not of class M. It appears to resemble the spectrum of R Andromedae, which is peculiar." On the Mount Wilson plates the spectrum closely resembles that of R Canis Minoris.

S Cassiopeiae

The bright enhanced iron lines λλ 4583, 4923, and 5018 are prominent on the second plate. They are slightly less intense on the third plate.

R Orionis

$H\alpha$ is a very intense emission line on Plate C 2424.

The type is not well marked. The spectrum appears to resemble that of R Canis Minoris.

H.D. 35155

This spectrum, from the violet to the red, closely resembles that of H.D. 22649, Boss 826. Plate C 672 is reproduced in Plate XII *d*, *Mt. Wilson Contr.*, No. 252; *Astrophysical Journal*, **56**, 457, 1922.

R Lyncis

The first four plates were taken with the 37½-inch reflector at Ann Arbor.[1]

The bright lines λ 4308 and λ 4571 are present on Plate C 2512. Traces of them are visible also on C 3539.

R Geminorum

Hydrogen lines.—$H\alpha$ is a strong emission line on Plates C 1402, C 1408, and C 3647. The plates listed in Table III show one or more hydrogen emission lines of shorter wave-length than $H\delta$.

The ratio of the intensity of $H\epsilon$ to that of $H\zeta$ is considerably greater on the last plate than on the preceding one (see discussion on p. 33 of similar behavior of the bright $H\epsilon$ line in the spectrum of U Cassiopeiae).

On Plates C 1339 and C 2480, taken fifty and fifty-eight days, respectively, before maximum, the hydrogen lines are comparatively weak; but the relative intensities of $H\beta$ and $H\gamma$ are approximately the same as on the other plates. From about phase −40 toward maximum the hydrogen lines brighten rapidly. Their intensities, however, do not closely follow the light-curve, but are greater after maximum than at the same number of days before maximum, and at a given magnitude the intensities are greater after maximum than before. In other words, there is a time lag in the behavior of the bright hydrogen lines compared with the total light.

[1] *Loc. cit.*

Miscellaneous bright lines.—λ 4521 appears as a bright line at about phase −30 and continues to +87 or longer. λ 4511 appears about −10, grows stronger, and becomes equal to λ 4521 at about +50. It is stronger than λ 4521 from then until +87, when both lines are weak.

λ 4308 appears as a bright line on practically all plates taken after maximum. λ 4202 also appears on the stronger plates. Both lines increase in intensity with phase, but λ 4202 is the weaker throughout. On Plate C 3694, phase +87, λ 4571 is a well-marked emission line. Bright lines were measured at λ 4215 and λ 4376 on the last two plates.

Absorption lines.—Very few absorption lines are well shown by the spectrograms, except near maximum. The continuous spectrum decreases in intensity so rapidly toward the violet that the region of satisfactory exposure is surprisingly short. Moreover, the lines have a tendency to become weak and indefinite when the star is faint. Thus it happens that on the present plates only a very few of the strongest lines can be followed for more than a few weeks at maximum. λλ 4535, 4554, and 4607 lie in a favorable region and are seen on practically all the spectrograms. λ 4335 is a blend of zirconium, titanium, and perhaps other lines; λ 4554 is the strongest ultimate line of the ionized barium atom; and λ 4607 is the well-known low-temperature line of strontium. λ 4554 remains throughout about twice as strong as λ 4535, but both are more intense near maximum. λ 4607, on the other hand, is decidedly weaker near maximum. The ratio λ 4607:λ 4554 shows a close correlation with the visual magnitude. Observations before maximum yield about the same curve as those after maximum. According to estimates with a low-power microscope, these two lines are equal when the magnitude is 7.8.

TABLE III

INTENSITIES OF BRIGHT HYDROGEN LINES

Plate	*Hβ*	*Hγ*	*Hδ*	*Hε*	*Hζ*	*Hη*	*Hθ*	*Hι*	*Hκ*
C 2003	25	15	7	.....	2	.....	.....	.....	.....
C 3097	25	12	6	.....	2	.....	.....	.....	.....
H 57	15	7	2	.....	1	.....	.....	.....	.....
H 61	30	15	6	.....	3	2	1	1	.....
C 3623	24	11	5	.....	1	.....	.....	.....	.....
C 3647	14	7	3	.....	0.5	.....	.....	.....	.....
C 3671	16	8	4	tr	1	.....	.....	.....	.....
C 3694	15	7	3	tr	tr	.....	.....	.....	.....

Plate C 133 is reproduced in Plate XII *b*, *Mt. Wilson Contr.*, No. 252; *Astrophysical Journal*, **56**, 457, 1922.

R CANIS MINORIS

Hα is an intense emission line on Plates C 2067, C 2568, and C 3615; it is visible on γ 9990 also, although this plate is underexposed.

This is not a typical spectrum of class S, and there may be some doubt whether or not the resemblance is sufficiently close to warrant its inclusion in this class. The chief objection is that the plates now available do not definitely show the zirconium bands in any portion of the spectrum. Were these bands present there would be no hesitation concerning its classification, as several other features of the spectrum have the same

characteristics as in class S. The spectrum of V Cancri closely resembles that of R Canis Minoris in the blue-violet portion, but in the red the zirconium bands are well marked. This circumstance favors including R Canis Minoris in class S. Comparison with UV Aurigae, 051532, on the other hand, is unfavorable to this classification, for the spectrum of this star at one time resembled that of R Canis Minoris, but at two other times appeared to be related to class N.

H.D. 62164

Remark in *H.D.*: "On an ordinary plate the spectrum consists of dark bands and is seen only in the region between $H\beta$ and $H\gamma$. On a plate stained with pinacyanol, the region of greater wave-length resembles class Mc. Perhaps the spectrum varies."

T Geminorum

Hydrogen lines.—The $H\alpha$ line is very weak or absent on C 3620. On Plate C 3672 the bright hydrogen lines have the following intensities: $H\beta$ 16, $H\gamma$ 9, $H\delta$ 6, $H\epsilon$ —, $H\zeta$ 2, $H\eta$ 0.5, $H\theta$ —.

Absorption lines.—λ 4535 has one-half or two-thirds the intensity of λ 4554. Near maximum, λ 4607 is about one-fourth as intense as λ 4554, but on Plates C 2072, C 2516, C 3171, and C 3186, taken about two months from maximum, it is nearly as strong as λ 4554. It is estimated that these lines become of equal intensity at mag. 10.0. All three lines appear weaker in this star than in R Geminorum.

Bands.—On Plate C 3620 the zirconium bands are strong in the red region; titanium bands are present but are less conspicuous. The titanium bands show a decided dependence upon phase, being much weaker at maximum than when the star is fainter. The zirconium bands show the same behavior to a lesser degree. Plate I *d* illustrates the change in the intensity of the bands.

H.D. 64332

Remark in *H.D.*: "The spectrum shows some resemblance to that of π^1 Gruis, R.A. 22^h16^m6, Dec. $-46°27'$, in the region between $H\beta$ and $H\gamma$. It appears to combine characteristics of classes M and R. Classified Ma on I 38629, on which it is very faint."

V Cancri

$H\alpha$ is a strong emission line on Plate C 3117.

The appearance of the spectrum on C 3673 is very different from that on the other plates. The intensity decreases toward the violet less rapidly than on C 2743, for example. From λ 4400 to $H\beta$ the spectrum is nearly continuous. All the lines are weak and have a curious blurred appearance. There are absorption lines in the position of $H\beta$ and $H\delta$. A defect in the emulsion makes the nature of $H\gamma$ indeterminate. *Ca* 4227 is a fairly narrow dark line.

Near time of maximum the spectrum in the blue and violet resembles that of R Canis Minoris, but in the red the zirconium bands are well marked, whereas in R Canis Minoris they are absent or very weak.

S Ursae Majoris

The first four plates were taken with the 37½-inch reflector at Ann Arbor.[1]

Hydrogen lines.—$H\alpha$ is a bright line of moderate intensity on Plate C 2068, but is

[1] *Loc. cit.*

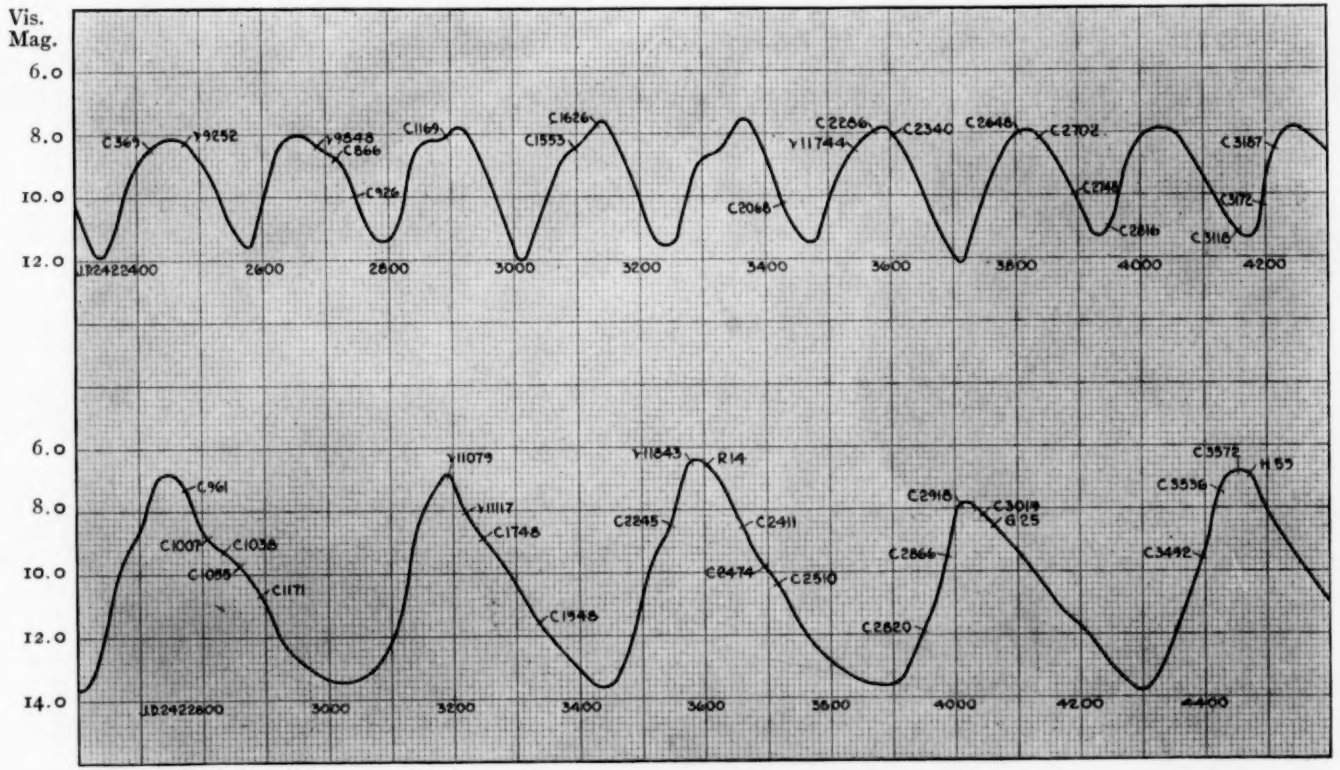

FIG. 1—Light-curves of S Ursae Majoris (above), and R Cygni (below)

not clearly seen on C 3118, although there is weak continuous spectrum in its vicinity. No hydrogen lines of shorter wave-length than $H\delta$ are visible on any of the plates. The bright hydrogen lines are absent at minimum, appear about phase -60, and increase in intensity rapidly toward maximum. After maximum they become less intense until at $+70$ they are again very feeble. They are probably present for a somewhat longer interval and to a slightly fainter magnitude after maximum than before, but the lag is not very marked. As shown by Table II, the relative intensities of the hydrogen lines are nearly the same on all plates except those taken in June, 1915, on which $H\gamma$ was evidently unusually strong.

Bands.—The zirconium bands are present throughout the light-cycle but are less intense at maximum than at other phases.

Figure 1 (upper curve) shows the light-curve of this variable and the distribution of my spectroscopic observations.

Plate C 373 is reproduced in Plate XII *c*, *Mt. Wilson Contr.*, No. 252; *Astrophysical Journal*, **56**, 457, 1922.

H.D. 121447

The band structure is slightly peculiar, not being identical with that in R Geminorum. This probably accounts for the zero estimate of the intensity of the zirconium bands on the third plate.

This spectrum resembles that of R Canis Minoris.

RW Librae

The S-type features are not well marked, and the classification is uncertain. The spectrum changes.

H.D. 156957

Remark in *H.D.*: "This spectrum appears to resemble those of π^1 Gruis and T Camelopardalis."

The Mount Wilson spectrograms were obtained by Mr. Sanford. See *Publications of the Astronomical Society of the Pacific*, **36**, 353, 1924.

H.D. 172804

The spectrogram was taken by Mr. Sanford. See *Mt. Wilson Contr.*, No. 276; *Astrophysical Journal*, **59**, 339, 1924.

R Cygni

The first three plates were taken by Wright with the 36-inch refractor at the Lick Observatory.[1]

Hydrogen lines.—$H\alpha$ is bright on all plates which show the red region. Bright $H\zeta$ is seen on two plates and has the following intensities: γ 11843, tr; H 59, 1. The bright hydrogen lines are very weak at phase -80, but strengthen rapidly toward maximum, after which they decrease in intensity. They are still of moderate intensity, however, at phase $+149$. Their behavior is thus unsymmetrical with respect to maximum. In this connection it must be borne in mind that the ascending branch of the light-curve is steeper than the descending. It is probable, however, that the bright hydrogen lines persist to a slightly fainter magnitude after maximum than before, but further observations are needed to determine definitely the characteristic times and magnitudes of appearance and disappearance.

[1] *Monthly Notices*, **72**, 548, 1912.

Miscellaneous bright lines.—λ 4521 appears in the spectrum about phase −45, followed twenty or thirty days later by λ 4511. λ 4511 slowly gains in relative intensity until it equals λ 4521 about phase +70. It is the stronger from that phase to +150 when both lines are weak.

The enhanced lines of iron, which are present in most Se spectra, are especially strong in R Cygni and persist throughout a considerable portion of the light-cycle. λ 4583 is, in fact, visible on practically all the Mount Wilson spectrograms having the proper exposure for this region. These extend from phase −80 to +149. On many plates λ 4583 is a conspicuous feature, while λλ 4549, 4924, 5018 are present with lower intensities. λ 4233 is faintly visible on two plates.

λ 4308 appears as an emission line on nearly all the plates taken after maximum, and λ 4202 on those taken after phase +60, but the intensity of λ 4202 does not exceed approximately one-half that of λ 4308 on any of my plates. The relative behavior is thus at variance with that in Me variables.[1] λ 4571 appears in emission at about +60 and soon becomes as intense as λ 4308. The following additional lines which have been observed in emission in Me variables have been measured as bright lines on two or more plates of R Cygni: λλ 4139, 4179, 4215, 4352, 4373, 4376, 4427, 4461. As far as may be judged from the small number of observations, their behavior is nearly the same as in the Me variables.

A curious phenomenon is the presence on the same plate of λ 4571 *Mg* and λ 4583 *Fe*+ as conspicuous bright lines. At about phase +100 they are of nearly equal intensity, λ 4571 being weaker before this epoch and stronger after it. λ 4571 is an extreme low-temperature line, while λ 4583 is a well-known spark line requiring high excitation for its production in the laboratory. The simultaneous occurrence of these two lines in a stellar spectrum is very puzzling. If they actually arise from the same region of the star, the physical conditions must differ in some important particular from those within the range of laboratory experience. It may be, however, that they come from different regions of the star, with low excitation for the magnesium line and high excitation for the iron line.

Absorption lines.—λ 4535 and λ 4554 appear more nearly equal than in R Geminorum, but prior to phase +30, λ 4554 is somewhat more intense. λ 4607 is much stronger than either of these lines except when the star is brighter than 7.4 mag.; it is estimated that λ 4607 and λ 4554 are of equal intensity at this magnitude, both before and after maximum.

Figure 1 (lower curve) shows the light-curve of this variable and the distribution of my spectroscopic observations. Plates γ 11117, C 1055, C 3492, and C 3572 are reproduced in Plate I.

Earlier observations.—In *Monthly Notices*, **72**, 546, 1912, Espin has given a summary of the early spectroscopic observations of R Cygni. The spectrum was generally assigned to Secchi's third type, and $H\beta$ was usually seen as a bright line. The Harvard observers noted that "the spectrum is peculiar, and shows changes on the various photographs." Espin's own observations during the autumn and winter of 1911 are quoted in detail. When the star was very faint, its spectrum was "apparently continuous," but when the magnitude reached 8.7, the bands were quite clear. Toward maximum the bands seemed less distinct. The Mount Wilson spectrograms do not afford indubitable evidence of systematic changes in the intensities of the bands, but they suggest, in harmony with

[1] *Mt. Wilson Contr.*, No. 200; *Astrophysical Journal*, **53**, 185, 1921.

Espin's observations, that the bands are slightly weakened when the star is brightest. Espin and one or two other observers thought the spectrum to be of type III (class M), but this is not surprising because the zirconium bands and the titanium bands present the same general appearance in the yellow and red. Espin saw the bright $H\beta$ line on several dates and thought it to be most conspicuous about two weeks before maximum. In the pages of the *Monthly Notices* following Espin's article, Wright gives an account of three spectrograms obtained at Lick Observatory shortly after the maximum observed by Espin. The spectrum seems to have been much the same as that shown by the Mount Wilson plates at the same phase.

AA CYGNI

Remark in *H.D.*: "The spectrum is faint, and it is uncertain whether it belongs to Class N or Class S."

On the Mount Wilson plates this spectrum offers a fine example of the presence of both titanium and zirconium bands in considerable intensity. Both sets of bands appear slightly stronger on the second plate, which is reproduced in Plate I *b*. A spectrum of class M5e (RS Ursae Majoris, 1925, April 9) is placed beside it for comparison. Notice the difference in structure near λ 4620.

Z DELPHINI

Remark in *H.D.*: "On a photograph taken November 20, 1903, the line $H\gamma$ is twice as bright as $H\delta$. The class of spectrum is not well defined. The brightest portion is between $H\beta$ and $H\gamma$."

A feeble bright $H\zeta$ line is present on Plate C 3493. $H\epsilon$ is not seen.

π^1 GRUIS

The spectrogram was taken at the Chile station of the Lick Observatory and loaned to me through the kindness of Dr. Campbell and Dr. Moore.

Additional notes on the spectra of most of these stars will be found in *Mt. Wilson Contr.*, No. 252; *Astrophysical Journal*, **56**, 457, 1922.

SUMMARY OF OBSERVATIONS

Zirconium bands.—The intensity of these bands is quite different in various S-type stars. In the variables they are present during all or most of the light-cycle, being slightly weaker near maximum. Their intensity is perhaps more closely correlated with magnitude than with the interval from maximum. In S Ursae Majoris they persist throughout the light-cycle, but in V Cancri they apparently disappear during the decreasing phase. More observations will be required to determine the characteristic behavior near minimum.

Titanium bands.—The intensity of these bands is widely different in various S-type stars. In about half of the variables they are absent or extremely weak at maximum and throughout the ob-

served phases, which in S Ursae Majoris include the entire light-cycle. In other variables they are present on all plates, but are fainter near maximum than at other times. In some stars the changes in intensity are very marked (see Plate I). In R Andromedae the titanium bands are weak or absent near maximum, but are strong when the variable is fainter.

The observations show very clearly that zirconium and titanium bands can exist simultaneously in a stellar spectrum. The significance of the relative behavior of these two groups of bands will be discussed below. One interesting observational feature which plays an important part in the recognition of the zirconium bands may be mentioned here. The titanium bands exercise a marked effect on a large portion of the spectrum, and, when strongly developed, practically obliterate the characteristic S-type structure between λ 4626 and λ 4700. A superficial inspection of the spectrograms might suggest that the zirconium bands disappear when those of titanium become strong, but a closer examination shows that in many cases the titanium band head at λ 4626 is apparently displaced about 6 A toward the violet from its normal position. This is caused by the fortunate circumstance that a zirconium band head occurs at about λ 4620.[1] It is this feature, illustrated in the spectrum of AA Cygni, Plate I *b*, which permits the detection of zirconium oxide when the titanium oxide bands are strong.

Hydrogen lines.—In non-variable S-type stars the Balmer lines are dark and of moderate intensity. The variable stars, with one exception, have exhibited very conspicuous bright lines near the maximum phase. The exception, AA Cygni, is a variable of class III with a small range and apparently irregular variation. Bright lines are not visible on the two Mount Wilson spectrograms and are not mentioned in the *Henry Draper Catalogue*. This star may thus be analogous to those irregular variables of class M, such as R Lyrae and α Herculis, whose spectra do not contain bright lines.

In the spectra of the long-period S-type variables, emission lines

[1] The identification is substantiated by unpublished laboratory observations by Dr. Kiess, of the Bureau of Standards, and by Dr. King, of the Mount Wilson Observatory. I am indebted to these gentlemen for the communication of their results in advance of publication.

of hydrogen appear during the time of increasing light at about one-fifth of the period before maximum. The variable at this time may be about midway between minimum and maximum on the ascending branch of the light-curve. The intensities increase rapidly so that at maximum the lines are very strong. They decrease again after maximum, but the interval of their visibility is greater than that before maximum, and the magnitude at which they finally disappear is probably slightly lower than at the time of their appearance before maximum. In these respects the bright hydrogen lines behave much as they do in Me variables.[1] One slight difference is that the phase displacement or time lag after maximum seems to be smaller in the S-type variables.

In spectra in which the titanium bands are absent, the intensities of the bright hydrogen lines decrease rapidly from $H\beta$ to $H\delta$ and the relative intensities remain about the same throughout the observed portion of the light-period. $H\alpha$ is a very strong line, probably stronger than $H\beta$. $H\zeta$ is frequently seen on strongly exposed plates, but $H\epsilon$ is usually absent or very weak. In U Cassiopeiae and R Geminorum, $H\epsilon$ has been observed to increase in relative intensity during decreasing light, due possibly to the diminishing absorption of the H line of calcium. Similar behavior of $H\epsilon$ is known in Me variables. In R Geminorum the regular decrease of intensity of the successive members of the Balmer series in the ultra-violet is interrupted by $H\iota$ which, as in class Me, is strong compared to $H\theta$ and $H\kappa$.

When titanium bands are present in the spectrum, the decrease from $H\beta$ to $H\delta$ is less rapid. $H\delta$ may equal or even exceed $H\gamma$ when the bands are strong.

Bright enhanced iron lines.—Several bright lines of ionized iron, particularly λλ 4583, 4924, and 5018, are present with various intensities in the spectra of nearly all Se variables. They are especially prominent in S Cassiopeiae and R Cygni. They are probably strongest near maximum, but their characteristic variation with phase is not clearly brought out by the present spectrograms. In the spectrum of R Cygni they persist through a considerable portion of the light-cycle. A remarkable fact is that at about phase +100, when

[1] *Mt. Wilson Contr.*, Nos. 200 and 311; *Astrophysical Journal*, **53**, 185, 1921, and **63**, 281, 1926.

the extreme low-temperature magnesium line λ 4571 has attained considerable strength, λ 4583 still remains a prominent feature of the spectrum. This is an interesting anomaly.

Miscellaneous bright lines.—The following bright lines, in addition to those of hydrogen and enhanced iron, have been observed in the spectra of one or more Se variables: λλ 4138, 4178, 4202*, 4215, 4308*, 4352, 4373, 4376, 4427, 4461, 4511*, 4521*, 4571*. All these lines are known in Me variables.[1] Those most frequently observed in the present investigation are marked by an asterisk and will be described in the following paragraphs. The general features of the behavior of the other lines, as far as they can be determined from the meager material, are about the same as in the Me variables. The line at λ 4215 seems to have a shorter wave-length than that measured by Joy at λ 4216.19 in *o* Ceti and may have a different origin.

λ 4308 appears at maximum, or soon after, and strengthens gradually as the light decreases. λ 4571 appears at one-tenth to one-fifth of the period after maximum and later becomes stronger than λ 4308. λ 4202 behaves about the same as λ 4308, but is less intense at all phases. λ 4308 makes its appearance rather earlier in the light-cycle in the Se variables than in those of class Me, and this is perhaps true of λ 4571 also. λ 4202, on the other hand, seems to come later, and is never so prominent as in Me variables shortly after maximum. The intensity of these three lines is not the same in all Se variables. They are weak, for example, in the spectrum of S Ursae Majoris.

The lines λ 4511 and λ 4521 have various intensities in different stars. They are fairly strong in the spectra of R Geminorum and of R Cygni, and weak in that of S Ursae Majoris. λ 4521 appears about one-tenth of the period before maximum (phase $-0.1p$). λ 4511 is first visible about $-0.04p$ and increases in relative intensity, becoming equal to λ 4521 at about phase $+0.15p$. It is the stronger from then on to $+0.3p$, when both lines are again weak. The behavior of these lines is thus much the same as in *o* Ceti,[2] except that in the Se variables they both appear somewhat earlier in the light-

[1] *Mt. Wilson Contr.*, Nos. 200, 265, and 311; *Astrophysical Journal*, **53**, 185, 1921, **58**, 195, 1923, and **63**, 281, 1926.

[2] Joy, *Mt. Wilson Contr.*, No. 311; *Astrophysical Journal*, **63**, 281, 1926.

cycle. The fact that λ 4511 has its maximum intensity at a later phase than λ 4521 probably indicates that the excitation most favorable for its production is lower than that for λ 4521.

Absorption lines.—The notes concerning the absorption spectrum of R Geminorum on page 35 apply to the other Se variables as well and explain why it is difficult to follow most of the absorption lines through any considerable portion of the light-cycle. Three lines only, λλ 4535, 4554, and 4607, have been selected for discussion in the following paragraphs. The probable identifications of these lines are stated on page 35.

The ratio of the intensity of λ 4535 to that of λ 4554 ranges, in different stars, from about 0.3 to 1.0. In the variables this ratio does not change markedly during the observed portion of the light-cycle, although in some cases it may decrease slightly near maximum.

The strontium line λ 4607 is generally one-half or two-thirds as intense as λ 4554 in the spectra of the non-variable S-type stars. In the variables its intensity varies with phase in a striking manner. At maximum it is much weaker than λ 4554 (in some instances only one-fifth as intense), but as the star becomes faint it develops rapidly and may become six to eight times as strong as λ 4554. It is also strong before maximum. In fact, as far as the present data go, the relation of the magnitude of the variable and the relative intensity of λ 4607 and λ 4554 seem to be the same for the ascending and the descending branches of the light-curve. Estimates of the magnitude at which λ 4554 and λ 4607 have equal intensities have been made in the case of four variables as follows:

	Mag. When λ 4554 = λ 4607	Max. Mag.	Mag. Difference
R Gem.....	7.8	6.9	0.9
T Gem.....	10.0	8.6	1.4
S Urs. Maj.....	8.7	7.9	0.8
R Cygni.....	7.4	7.0	0.4

The maximum magnitude in the third column is the mean of the magnitudes at the maxima which occurred during the period of my spectroscopic observations.

DISCUSSION

The present observations fully support the previous conclusion that S-type spectra are closely related to those of giant stars of class M.[1] In investigating the physical conditions prevailing in S stars it will therefore be natural to study the relationship between the two types of spectra.

The first thing to notice is the relative behavior of the zirconium and titanium bands. In the ordinary M-type spectra, titanium bands alone are seen; in some S-type spectra we find only the zirconium bands; while in certain other spectra both kinds of bands are present. The behavior of the bands in variable S-type stars is outlined on pages 40 and 41.

The stellar observations indicate that in general the temperature range for the production of the zirconium bands is higher than that for the titanium bands, and this is borne out by King's experiments with the electric furnace, which showed that "the zirconium bands belong to the oxide and their presence indicates a distinctly higher temperature stage than that required for the bands of titanium oxide."[2] The laboratory work does not give a definite upper limit for the production of the bands, but it is probably higher for zirconium than for titanium.

The temperatures of the reversing layers of most S-type stars seem to be near the upper limit for the production of titanium bands. Under the prevailing conditions these bands evidently strengthen rapidly with comparatively small decreases in temperature, and we may therefore arrange the stars in a sequence of decreasing temperature according to the intensity of the titanium bands. Such an arrangement implies that the titanium-band intensity is a function of temperature alone, and, while this may not be strictly true, the evidence indicates that it is at least approximately so. The changing temperatures of several variable stars and the relationships to the titanium bands are shown diagrammatically in Figure 2.

For estimating the relative temperatures of M- and S-type stars the following means are available: (1) color; (2) heat index from measurements of total radiation; (3) absorption-line spectrum;

[1] If dwarf stars of class S exist, they are not yet known.

[2] *Publications of the Astronomical Society of the Pacific*, **36**, 140, 1924.

(4) bands, especially those of titanium; (5) behavior of bright lines in variable stars. These indicate that the temperatures of various S-type stars cover a range which overlaps that of the M-type stars, and it thus seems clear that the difference between the two types is not solely one of temperature.

Three important factors, other than temperature, in which stars may differ are mass, density, and chemical composition. Difference in mass between M- and S-type stars is possible, but we have no direct means of establishing it or of proving that it would produce the observed spectral differences. The meager evidence available

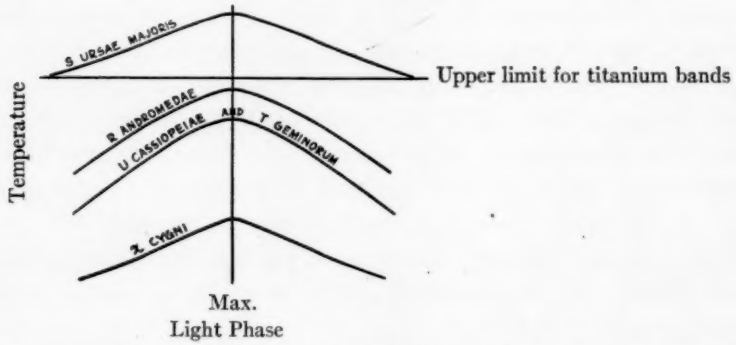

FIG. 2.—Diagram illustrating behavior of titanium bands in S-type variables

suggests equal or larger masses for the S-type stars. S-type stars may contain more zirconium and barium than those of class M, but this possibility should be appealed to only as a last resort, because we are not now prepared to deal effectively with the only problems which it suggests. We are left, then, with difference in density as the most promising subject of inquiry, and here a positive result seems attainable.

A lower density for S-type stars is indicated by the greater intensity of enhanced lines. The most outstanding cases in the absorption spectrum are the strontium lines λ 4077 and λ 4215. The H and K lines of calcium are, I think, stronger in S-type spectra. Barium λ 4554 is much stronger in class S, but this line may possibly be weakened in class M by some cause not directly connected with density. Bright enhanced lines of iron appear in the spectra of

most S-type variables. According to a suggestion by King, based on his observations of the high-current arc in vacuum,[1] low density may be an important factor in producing the curious combination of enhanced lines with low-temperature lines in S-type spectra. The general appearance of the typical S-type spectrum is that associated with high luminosity and low density. The S-type stars may bear somewhat the same relationship to those of class M that Cepheids and pseudo-Cepheids do to ordinary F and G stars. The fact that S-type stars exhibit some tendency to congregate near the galactic plane, while the M-types, in general, are practically free from this tendency, is in line with this idea.[2]

The relatively long periods of the S-type variables constitute another strong argument for low density. The temperatures of most S-type stars appear to approximate those of types M1–M3, but the periods correspond to those of variables of types M6–M8, which are generally believed to have lower densities.

A possible additional argument for the low densities of S-type variables lies in the fact that their velocities are about the same as those of classes M6–M8e,[3] the most tenuous of the M-type variables. The relatively small velocities may also indicate higher masses.

No observational fact seems inconsistent with the idea of lower density of S-type stars, and several features strongly favor it. We may therefore accept it as a working hypothesis of the physical difference between the two closely related classes of red stars. It seems possible that the only essential difference between classes M and S is one of density, but judgment must be reserved on this point. It would be extremely interesting if laboratory investigation should show that, in a mixture of titanium oxide and zirconium oxide vapors, low density is relatively favorable to the zirconium bands.

A few remarks concerning certain problems of red stars, suggested by the comparison of S-type spectra with those of class M,

[1] *Ibid.*, **38**, 235, 1926.

[2] Adams, Joy, and Humason have recently called attention to a noticeable galactic concentration of supergiant stars of class M (*Mt. Wilson Contr.*, No. 319; *Astrophysical Journal*, **64**, 225, 1926).

[3] Table XII, *Mt. Wilson Contr.*, No. 264; *Astrophysical Journal*, **58**, 215, 1923.

will be made in conclusion. Similarities and differences brought out by this comparison have been described in the preceding pages.

Numerous spectral features common to the two types, or accentuated in the M-types, may be accounted for by low temperature; and certain differences find an explanation in the hypothesis that S-type stars have lower densities than M-type stars at the same temperature.[1] Several remarkable features observed only in long-period variables exhibit similar behavior in classes M and S, and indicate that the circumstances accompanying the variation (and probably its fundamental cause) are the same in both classes. The presence of titanium oxide is evidently not a necessary condition for these phenomena.

Certain other effects, however, seem to be closely correlated with the intensity of the titanium bands. In general, these effects consist of the surprisingly low intensities of certain features which might be expected to be strong. To be specific we may mention (1) weakness of bright $H\alpha$, $H\beta$, and $H\gamma$ in comparison with $H\delta$ and the ultraviolet Balmer lines; (2) weakness of certain ultimate absorption lines, notably D_1, D_2 of sodium, and λ 4607 of strontium; (3) weakness of the continuous spectrum from λ 4200 toward longer wavelengths, often made evident by the surprisingly great relative intensity in the region λ 4100 to λ 4200.

Low temperature will not directly account for these peculiarities, and a special cause must be sought. It is perhaps to be found in the absorption of the titanium oxide bands, as in both M- and S-type stars the effects become more marked with increase in intensity of the bands.

The titanium oxide, if responsible for the weakness of the spectral features under discussion, must lie at a higher level than that in which these are produced, and must exert a powerful absorption on the regions of the spectrum in which they are located. If light-pressure is proportional to the amount of radiant energy absorbed, then titanium oxide receives far greater upward impulses than any other substance known in the atmospheres of M- and S-type stars, and may thus exist at a very high level. Moreover, it is reasonable

[1] An alternative statement of the hypothesis is that S-type stars have higher temperatures than M-type stars of the same density.

to believe that when the bands appear very strong, the titanium oxide layer is nearly opaque in certain spectral regions (including those of the weakened features under discussion) and practically shuts off radiation from below, the energy which we observe in these regions being largely that emitted by the titanium oxide itself. This would explain how a spectral line can be eliminated from a region in which strong continuous spectrum is still observed.

The deletion of the bright $H\epsilon$ line by the absorption of the overlying calcium can be explained in a similar way. It has been difficult to understand how the hydrogen line could suffer greater absorption than the continuous spectrum which can be photographed within the broad H line, but if the calcium is quite opaque in this region, the light observed there being emitted by the calcium itself, this difficulty is overcome.

The attempt to explain the abnormal relative intensities of the bright hydrogen lines in M-type variables by the absorption of titanium oxide will meet serious, but perhaps not insuperable, difficulties. Relative motion of the effective levels of hydrogen emission and titanium absorption may be a part of the explanation. Measurements of the displacements of the bright lines show the hydrogen apparently moving upward with respect to the absorbing materials at the time of maximum. If the effective levels of the hydrogen emission lie partly within the stratum of absorbing titanium oxide, this relative motion may conceivably explain the fact that the weakened lines are stronger after maximum than before.

The value of S-type stars in testing this and other hypotheses concerning red stars lies in the fact that their temperature range overlaps that of the M stars; we are thus offered the opportunity of distinguishing between the general effects of low temperature and special effects such as those of low density or of the absorption of titanium oxide. Class N also may furnish useful comparisons, but its relationship to other classes of red stars is not so close as that between classes M and S.

MOUNT WILSON OBSERVATORY
August 1926

AN APPLICATION OF THE QUANTUM THEORY TO THE LUMINOSITY OF DIFFUSE NEBULAE

By H. ZANSTRA[1]

ABSTRACT

H. N. Russell, on theoretical grounds, made the suggestion that the *emission spectrum* observed *in nebulae* is, in some way, *induced* by *radiations from neighboring stars of a high temperature. This conclusion was reached independently* by E. P. Hubble, who, by an extended study of *observational data, settled this question* beyond any reasonable doubt. He also found a number of *qualitative and quantitative relations to exist* between the *characteristics of the nebula and of the star involved in its luminosity.* In line with this idea, already widely accepted, the writer has aimed to work out a *quantitative treatment* of the *emission spectrum produced in diffuse nebulae,* using Hubble's quantitative results as a guide. The problem is simplified by assuming that the star emits radiation like a black body and that the nebula consists of hydrogen of which an appreciable amount is in the atomic form. It is assumed that the ultra-violet starlight beyond the head of the Lyman series is completely absorbed by the nebula, causing ionization of the nebular hydrogen atoms into protons and free electrons. The subsequent recombination of electrons and protons causes the re-emission by the nebula of a number of line spectra and continuous spectra, among others the Balmer series and the continuous spectrum at its head. Finally, an *approximate expression for the ratio of the photographic intensity of the nebula to the photographic intensity of the intercepted starlight* is obtained, and this ratio is calculated for different temperatures (T) of the star; it is unity for $T = 33{,}000°$. Hubble found this ratio to be approximately unity for stars of type Bo and O associated with diffuse nebulae, and hence the *theory requires for those stars an average temperature of about 33,000° C.* A more detailed discussion of the observational data leads to the following rough values for the *temperature of stars associated with diffuse nebulae:* B1, 21,000°; Bo, 28,000°; O, 34,000°. Stars in planetary nebulae would, in general, have a still higher temperature. These values are in fair agreement with the temperature determinations from the theory of thermal ionization of stellar atmospheres, and this agreement serves as a check for the mechanism of luminosity proposed. The fact that the continuous spectrum at the head of the Balmer series is more conspicuous in planetaries than in diffuse nebulae led to the assumption that free electrons of higher velocity are captured to a large extent by the lower levels of the hydrogen atom. This offers an explanation for the result of H. H. Plaskett in work not yet published, who finds that the first lines of the Balmer series in nebulae become relatively stronger, as the temperature of the associated star becomes higher. The assumption is also in harmony with the theoretical considerations of H. A. Kramers.

1. BRIEF REVIEW OF OBSERVATIONAL FACTS AND THEORETICAL IDEAS REGARDING GALACTIC NEBULAE

The galactic nebulae are usually subdivided into two classes, diffuse and planetary nebulae.

The planetary nebulae have a definite structure, appearing as

[1] National Research Fellow. The present discussion of nebular luminosity was, in its main features, given in a paper read before a meeting of the American Physical Society, under the same title (see *Physical Review*, **27**, 644, 1926).

ovals, rings, or disks. Their spectrum consists of a number of single lines, and frequently there is also a continuous spectrum, starting approximately at the head of the Balmer series of hydrogen and extending into the ultra-violet. Following Hubble, a spectrum of this type will be classified as an "emission" spectrum. Each planetary nebula has a star approximately at its center.

The diffuse nebulae, on the other hand, do not show a definite sharp structure. Hubble divides them into dark nebulae, obscuring the background, and luminous nebulae. The luminous nebulae have either emission spectra, similar in a general way to those of the planetaries, or continuous spectra of approximately the same type as, although less intense than, those of ordinary stars. Frequently the three types of diffuse nebulosity, "dark," "continuous," and "emission," are found in the same regions of the sky and even merge more or less into one another, as for example in the large Orion nebula.

It is generally recognized at present that both the continuous and the emission spectrum observed in nebulae are, in some way, caused by neighboring stars.

In a number of cases V. M. Slipher[1] has found that the continuous spectra of certain nebulosities are approximately the same as those of stars in their neighborhood and suggests, for this reason, that those nebulae owe their luminosity to reflected starlight. E. Hertzsprung[2] estimated that the luminosity of the Pleiades nebulae is from 3.5 to 5.6 mag. fainter than would be the case for a perfect diffuse reflection, although a more recent statistical study of a large amount of observational material by Hubble leads to a much stronger luminosity for diffuse nebulae in general.

If the nebular spectrum is of the emission type, an explanation of the luminosity by simple reflection of starlight is out of the question, since the nebular spectrum is then entirely different from that of the star. H. N. Russell[3] has suggested that such emission nebulae are excited in some way by radiation, either short wave-length or

[1] *Publications of the Astronomical Society of the Pacific*, **30**, 63, 1918.

[2] *Astronomische Nachrichten*, **195**, 448, 1913.

[3] *Observatory*, **44**, 72, 1921; *Proceedings of the National Academy of Sciences*, **8**, 115, 1922.

corpuscular, the most reasonable source of such radiation being a very hot body, such as a star of early type.

An extended study by E. P. Hubble,[1] based on a large amount of observational material, establishes beyond any reasonable doubt that the luminosity of the different types of galactic nebulae is due to neighboring stars. In the first of his two papers a classification of nebulae is given, and he finds that with practically every luminous nebula a star is associated which can be held responsible for the nebular luminosity. There is a definite relation, which holds with very few exceptions, between the spectral characteristics of this star and of the nebulosity; stars of type B1 and later are usually associated with diffuse nebulosity of the continuous type, stars of type Bo and Oe5 with diffuse nebulosity of the emission type, and still earlier stars with planetary nebulae.

In the second paper a quantitative relation between the luminosity of a nebula and the luminosity of the associated star is derived by a statistical study of observational data.

This relation may be formulated as follows: (*a*) The light emerging from a patch of diffuse nebulosity having a continuous spectrum is the equivalent of the starlight intercepted (star type B1 and later).[2] (*b*) The light emerging from a patch of diffuse nebulosity having an emission spectrum is the equivalent of the starlight intercepted (stellar type notably Bo and Oe5). (*c*) The light emerging from a part of the nebulosity in a planetary nebula is, on the average, 4 to 5 mag. brighter than the equivalent of the starlight intercepted (type of central star earlier than Oe5).

Quantities of light are called "equivalent" if they produce the same blackening on the photographic plate under the conditions of observation.

This empirical relation will be our guide in discussing the theoretical mechanism by which the luminosity of the nebula may be caused.

[1] *Astrophysical Journal*, **56**, 162, 400, 1922.

[2] Hubble's interpretation implies that the light emerging from the nebula is reemitted evenly in all directions. A different result will therefore be obtained, if the continuous spectrum is assumed to be due to reflection of the starlight by the nebula. This question will be discussed in sec. 4.

2. LUMINOSITY PRODUCED IN A NEBULA CONSISTING OF HYDROGEN BY A STAR EMITTING BLACK-BODY RADIATION

Attention will be confined mainly to the production of an emission spectrum in the nebula. Apparently there are no cases known where a normal star of type B1 or later produces a nebular emission spectrum of appreciable intensity. The hotter stars of type Bo and Oe5 are able to produce a strong nebular emission spectrum. The stars in planetary nebulae, which have a higher temperature still, produce also a more intense nebular spectrum. This strongly suggests that the temperature of the associated star is the main factor in determining the intensity of the nebular emission spectrum.

In the discussion which follows the star will be replaced by a black body of surface temperature T. If the star be a sphere of radius R, the total radiation per second within the frequency interval $d\nu$ is

$$\pi R^2 c\rho_\nu d\nu , \qquad (1a) \qquad \rho_\nu = \frac{8\pi h}{c^3} \cdot \frac{\nu^3}{e^{\frac{h\nu}{kT}} - 1} , \qquad (1b)$$

$$h = 6.54 \times 10^{-27} , \quad k = 1.372 \times 10^{16} ,$$

the formula (1b) being Planck's formula for the energy density $\rho_\nu d\nu$ of black-body radiation.[1]

The total number of quanta emitted per second within $d\nu$ is obtained by dividing this by $h\nu$ and therefore becomes

$$\frac{8\pi^2 R^2 k^3}{c^2 h^3} T^3 \frac{x^2}{e^x - 1} dx , \qquad (2) \qquad x = \frac{h\nu}{kT} . \qquad (2a)$$

If Ω be the solid angle subtended by a patch of nebulosity from

[1] Well-known considerations of radiation equilibrium lead to the following result: The energy within $d\nu$ emitted by an area $d\sigma_1$ of a black body of temperature T and intercepted by a surface element $d\sigma_2$ at distance r is $I_\nu \, d\nu \, d\sigma_{1n} \, d\sigma_{2n}/r^2$, where $I_\nu = \rho_\nu c/4\pi$ and $d\sigma_{1n}$ $d\sigma_{2n}$ are the projections of $d\sigma_1$ and $d\sigma_2$ on a plane perpendicular to r. The formula (1a) for the radiation emitted by a spherical body is easily obtained from this by considering as σ_1 the surface of the black body and as σ_2 a large concentric sphere, through which the radiation passes outward.

the center of the star, the number of quanta within $d\nu$ intercepted per second by this patch is

$$\Omega \frac{2\pi R^2 k^3}{c^2 h^3} T^3 \frac{x^2}{e^x - 1} dx , \tag{3}$$

as is found by multiplying (2) by $\Omega/4\pi$.

The number of intercepted quanta which, under observational conditions, can effect the photographic plate will be taken as a rough measure of the photographic intensity of the intercepted starlight. The long wave-length limit of an ordinary photographic plate is about 5050 A, that for the atmospheric absorption is about 3300 A corresponding to the frequencies $\nu_1 = 5.95 \times 10^{14}$ per second and $\nu_2 = 9.10 \times 10^{14}$ per second. If a reflector is used, only quanta having frequencies between those two limits can affect the photographic plate. The number of such "photographic quanta" in the starlight intercepted by the patch of nebulosity per second is, therefore,

$$N_{ph} = \Omega \frac{2\pi R^2 k^3}{c^2 h^3} T^3 \int_{x_1}^{x_2} \frac{x^2}{e^x - 1} dx , \quad (4) \qquad x_1 = \frac{h\nu_1}{kT} , \quad (4a) \qquad x_2 = \frac{h\nu_2}{kT} , \quad (4b)$$

which is an approximate measure of the photographic intensity of the starlight intercepted.

The mechanism by which the radiation intercepted by a patch of nebulosity can be transformed into an emission spectrum will now be considered. The observed emission spectra, in which the Balmer series of hydrogen is very conspicuous, indicate that the diffuse nebulae are composed of certain gases, of which atomic hydrogen is the most active in producing the luminosity. Therefore the problem will be simplified by assuming the nebula to consist wholly of hydrogen, an appreciable amount of which is atomic. Such a nebula may absorb a part of the radiation which it intercepts, and for this only atoms in the normal state need to be considered.[1] The quantum theory provides two mechanisms by which this absorption may occur. In the first place, a hydrogen atom in its normal state may

[1] Because the incident radiation has a very low density, it may reasonably be expected that the fraction of neutral atoms in the excited states is negligible, although it must be admitted that the present knowledge of the statistics of the atom is insufficient to decide this question.

absorb the different frequencies of the Lyman series, and the atom thus excited will subsequently return to its normal state, either directly or in steps, the electron falling back to the different energy-levels before it reaches the lowest level. In doing this it re-emits a number of line spectra, among others the Lyman series and the Balmer series. The mechanism of ordinary excitation accounts in this way for some luminosity in the nebula. This effect, however, is much too weak to account for the total luminosity observed, and it will not be further considered.

The second, and much the more powerful, mechanism which the quantum theory provides is that of ionization and subsequent recombination. An atom in its normal state is capable of absorbing all the energy of a wave-length shorter than the head of the Lyman series, frequency $\nu_0 = 32.84 \times 10^{14}$ per second. The number of quanta per second thus absorbed from the ultra-violet starlight by a patch of nebulosity is, in view of (3),

$$N_{ul} = \Omega \frac{2\pi R^2 k^3}{c^2 h^3} T^3 \int_{x_0}^{\infty} \frac{x^2}{e^x - 1} dx , \qquad (5) \qquad x_0 = \frac{h\nu_0}{kT} , \qquad (5a)$$

provided that the absorbing layer contains a sufficient number of normal atoms for the absorption to be practically complete.

For each quantum $h\nu$ thus absorbed, an electron, mass m, is knocked out of the atom and becomes a free electron with the velocity v given by the photo-electric equation

$$h\nu = W + \tfrac{1}{2} m v^2 , \qquad (6)$$

where $-W = -W_1 = h\nu_0$ is the energy of the first atomic level. These free electrons will recombine with the nuclei by falling back to one of the different levels, energies $-W_1$, $-W_2$, $-W_3$, etc., emitting the continuous spectrum at the head of the Lyman series, at the head of the Balmer series, at the head of the Paschen series, etc., according to the same equation (6). Subsequently the electrons on the higher levels 2, 3, 4, etc., will return to the normal level 1, either directly, under emission of the Lyman series, or in steps, emitting the other hydrogen lines, among others the Balmer series. In this way a number of continuous spectra and line spectra will be re-emitted,

and this would account for the Balmer series in the nebular spectrum as well as the continuous spectrum sometimes observed at its head.

The next question is the intensity of the re-emitted spectra. To solve this completely would require a complete knowledge of the probabilities of recombination with the levels 1, 2, 3, etc., as well as of the probabilities of the transitions in the hydrogen atom, or at any rate of their ratios. Without going into details, however, a few general statements can be made. Any ionized atom will have to return, sooner or later, to its normal state, where the electron is in its first energy-level. The very last electron jump is therefore either a jump of a free electron to this first level or a jump from one of the higher levels to this first level, the corresponding frequencies emitted being those of the continuous spectrum at the head of the Lyman series and the lines of the Lyman series. Assuming a stationary state of the nebula, viz., that just as many atoms become ionized per unit time as return from the ionized to the normal state, then, since the number of electrons emitted photo-electrically from the first level is equal to the number of ultra-violet quanta absorbed, N_{ul}, the number of electrons falling back to the first level is also N_{ul}. Hence the number of quanta re-emitted by the nebula, either as the continuous spectrum at the head of the Lyman series or as lines of the Lyman series, is equal to N_{ul}, or symbolically

$$Ly_c + Ly = N_{ul} , \tag{7}$$

where Ly_c and Ly represent the numbers of quanta of the two different kinds.

There is reason to believe, however, that practically all the Lyman quanta will ultimately leave the nebula as the first line of the Lyman series $Ly\alpha$. To fix the ideas, take the line $Ly\beta$ as an example. This line, after being produced for the first time, can, on its way out of the nebula, excite another hydrogen atom to the level 3; this second atom may then either re-emit the first line of the Balmer series $H\alpha$ and subsequently $Ly\alpha$, or it may re-emit $Ly\beta$. In the second case the quantum $Ly\beta$, on its way out of the nebula, may again be absorbed by another hydrogen atom and be re-emitted, either as a quantum $H\alpha$ and a quantum $Ly\alpha$, or as the quantum $Ly\beta$. If this secondary absorption and re-emission goes on a sufficient

number of times, all the original radiation $Ly\beta$ will have been split up into $H\alpha$ and $Ly\alpha$. In other words, the splitting up of all quanta $Ly\beta$ into $H\alpha$ and $Ly\alpha$ will be complete, if the absorption coefficient of $Ly\beta$ is sufficiently large compared with the original absorption of the continuous spectrum. This condition is presumably fulfilled, because the absorption coefficient for a single spectral line may in general be expected to be much larger and perhaps even of an entirely different order than the absorption coefficient for the continuous spectrum at the head of the series. Carrying out a similar reasoning for the lines $Ly\gamma$, $Ly\delta$, etc., we arrive at the conclusion

$$Ly = Ly\alpha \,. \tag{8}$$

The next point of interest is the Balmer series and the continuous spectrum at its head. These are produced, if an electron falls back to the level 2, a process which always precedes the production of one quantum $Ly\alpha$. Hence the number of quanta of these two kinds Ba_c and Ba is equal to the number of quanta $Ly\alpha$.

$$Ba_c + Ba = Ly\alpha \,.$$

This, together with (7) and (8), leads to

$$Ly_c + Ba_c + Ba = N_{ul} \,. \tag{9}$$

Therefore a number of quanta N_{ul} emerge from the patch of nebulosity, partly as Ly_c, partly as Ba_c, and partly as Ba.

Only those quanta with frequencies in the interval $\lambda_1 = 5050$ A to $\lambda_2 = 3300$ A can affect the ordinary photographic plate under observational conditions. Hence the quanta Ly_c and the quanta of the first line of the Balmer series $H\alpha$ do not affect the photographic plate, but the remaining quanta Ba_c and the quanta of the other lines of the Balmer series $H\beta$, $H\gamma$, $H\delta$, etc., are all "photographic quanta." The number of these emerging from the nebula is therefore

$$N'_{ph} = N_{ul} - (Ly_c + H\alpha) \,, \tag{10}$$

which is smaller than the number of absorbed ultra-violet quanta by the amount $Ly_c + H\alpha$.

The line $H\alpha$ in the nebular spectrum, taken with panchromatic plates, though it is strong, shows an intensity comparable with the

other lines $H\beta$ and $H\gamma$. The continuous spectrum at the head of the Lyman series, arising from the capture of free electrons by level 1, is not accessible to direct observation, but it is reasonable to expect that its intensity in quanta is of the same order as that of the continuous spectrum Ba_c at the head of the Balmer series, which, in a similar way, is produced by the capture of free electrons by level 2. The spectrum Ba_c in diffuse nebulae, as far as is known, has only been observed in one case.[1] For planetary nebulae, where the temperature of the associated star is much higher, this spectrum Ba_c becomes more conspicuous. It seems safe, therefore, to assume that, for diffuse nebulae, the quantity Ba_c, and, by indirect argument, the quantity Ly_c is either negligible or of the same order as the number of quanta in the lines $H\beta$ and $H\gamma$. The physical nature of this assumption is probably that electrons of rather small velocity, as produced by a star of lower temperature, are mostly recaptured on the higher levels of the hydrogen atom, whereas, for electrons of higher velocity, freed in the planetary nebulae, the chance of being captured by the lower levels becomes much larger. On this picture, the nebular spectrum approaches more to a pure line spectrum as the temperature of the associated star gets lower, which is in line with the observational evidence. The considerations just given indicate that, for diffuse nebulae at any rate, the quantity Ly_c+Ha occurring in equation (10) is of the same order as the quantity N'_{ph}, and, therefore, as regards order of magnitude

$$N'_{ph} \sim N_{ul} . \tag{11}$$

Combining (11) with (4) and (5), the ratio of the number N'_{ph} of photographic quanta emerging from a patch of nebulosity to the number N_{ph} of photographic quanta in the intercepted starlight is found to be approximately

$$\frac{N'_{ph}}{N_{ph}} \sim \frac{N_{ul}}{N_{ph}} = \frac{\int_{x_0}^{\infty} \frac{x^2}{e^x-1}\,dx}{\int_{x_1}^{x_2} \frac{x^2}{e^x-1}\,dx} , \tag{12}$$

$$x=\frac{h\nu}{kT} , \qquad \nu_0=32.84\times10^{14} , \qquad \nu_1=5.95\times10^{14} , \qquad \nu_2=9.10\times10^{14} .$$

[1] E. P. Hubble, *Publications of the Astronomical Society of the Pacific*, **32**, 155, 1920.

This is also a rough measure of the ratio L of the photographic intensity of the emission spectrum of a patch of nebulosity to the photographic intensity of the intercepted starlight.

If quanta of an entirely different frequency were compared, the assumption that the number of quanta between the two assigned limits is a measure of photographic intensity would not be allowed, even as an approximation. But in the final equation (12) the inaccuracy introduced by this assumption of "photographic quanta" is only due to the difference in distribution of intensity of the continuous spectrum of the star as compared with the nebular emission spectrum, both covering a large range of frequency, and here the procedure followed, although very inaccurate, should give the right order of magnitude for the luminosity ratio L.

The values of the integral occurring in (10), taken between the limits x and ∞, are given in Table I for different values of x. The integrals between subsequent values of x were obtained graphically with an accuracy of at least 1 per cent, except for the first two intervals 0–0.2 and 0.2–0.3, where the accuracy is 2 per cent. For values of x larger than 6 the numerator in the integral was replaced by e^x and the integral solved by the ordinary process; the accuracy is then at least $\frac{1}{4}$ per cent, since $e^6 = 403.4$. In the table, only four decimals were given.

With the aid of Table I the values of $\int_{x_0}^{\infty}$ and $\int_{x_1}^{x_2}$ were determined by graphical interpolation for a series of temperatures T from 15,000° up to 200,000°. The values of those integrals and their ratio $\int_{x_0}^{\infty} \Big/ \int_{x_1}^{x_2}$ are given in Table II. The last column of Table II contains the value of this ratio, expressed in magnitudes.

Table II shows that the ratio L is a function of the temperature T of the associated star which increases very rapidly with T, hence if L is known from observations, the temperature of the associated star required by the mechanism of ionization and recombination may be calculated. For diffuse nebulae with emission spectra, where the stars are predominantly of the types Bo and Oe5, Hubble found empirically that the nebular luminosity is the equivalent of the inter-

cepted starlight, and hence $L = 1$ (sec. 1,*b*). The corresponding temperature, as found from Table II, is approximately 33,000°. To account for the nebular luminosity observed, the theoretical mechanism thus requires an average temperature of 33,000° for Bo and O

TABLE I

VALUES OF $\int_x^\infty \frac{x^2}{e^x - 1} dx$

x	$\int_x^\infty$	x	$\int_x^\infty$	x	$\int_x^\infty$
0.0.....	2.4002	2.....	1.4176	7.....	0.0593
0.2.....	2.3824	3.....	0.8634	8.....	.0275
0.3.....	2.3606	4.....	.4792	9.....	.0125
0.6.....	2.2562	5.....	.2496	10.....	0.0055
1.....	2.0516	6.....	0.1240		

TABLE II

T	$\int_{x_0}^\infty$	$\int_{x_1}^{x_2}$	Ratio $\int_{x_0}^\infty \Big/ \int_{x_1}^{x_2} \omega L$	Ratio in Magnitudes
15,000°.....	0.0043	0.57	0.0075	5.31
20,000.....	.032	.48	.066	2.95
25,000.....	.103	.38	.271	1.42
30,000.....	.216	.30	0.72	0.36
35,000.....	.35	.25	1.41	− .37
40,000.....	.50	.20	2.50	−0.99
50,000.....	0.80	.150	5.4	−1.83
70,000.....	1.27	.083	15.3	−2.96
100,000.....	1.68	.044	38	−3.95
150,000.....	2.02	.020	101	−5.01
200,000.....	2.16	0.0115	185	−5.68

T is the temperature of the star producing the luminosity. The ratio in the fourth and fifth columns represents the approximate theoretical value for the ratio *L* of the photographic intensity of the nebula to the photographic intensity of the starlight intercepted.

stars associated with diffuse nebulae. This value of the temperature seems quite reasonable, and serves as a first check of the proposed mechanism of luminosity. In the next section, a more detailed discussion of the observational data will be given.

3. COMPARISON WITH OBSERVATION; TEMPERATURE OF BO AND O STARS

Hubble's method for obtaining the empirical value of *L*, the ratio of nebular luminosity to starlight intercepted, involves the

photographic magnitude m of the star and the maximum angular extent a_1 of the nebular luminosity on the photographic plate for an exposure of one hour. Assuming that the light re-emitted by the nebula is the photographic equivalent of the starlight intercepted, Hubble[1] derived the following expression:

$$m+5 \log a_1 = 11.61 ,$$

by spreading the luminosity of the star over a spherical shell of a radius a_1, on the implied assumption that the light is re-emitted by the nebula evenly in all directions. He corrects for the fact that the line connecting the star with the extreme point of the nebulosity is not perpendicular to the line of sight, by subtracting 0.98 mag. from the right-hand side of this equation. We find, however, that this correction amounts only to 0.52 mag.[2] Hence we shall take

$$m+5 \log a_1 = 11.09 , \qquad (13)$$

if the light re-emitted by the nebula is the photographic equivalent of the intercepted starlight.

In the observational material contained in Table I of Hubble's second paper, there are ten O stars (seven stars classed as Oe5, one star Ob, and two stars Odp) and five Bo stars connected with diffuse-emission nebulae.

[1] *Astrophysical Journal*, **56**, 410, 1922.

[2] Assuming different directions of the line r connecting star and extreme nebulous patch to be equally probable, the average projection of r perpendicular to the line of sight is $\frac{\pi}{4}r$. This may be taken into consideration by adding $5 \log \frac{\pi}{4} = -0.52$ to the right-hand side of the equation. The value of $\frac{\pi}{4}r$ is obtained in the following way. The star may be considered as the center of a sphere of radius r, and the patch of nebulosity is then at the surface of the sphere. For the position of the nebular patch, equal areas $d\sigma$ of the sphere are equally probable. The line of sight being fixed, then if the angle between r and the line of sight be ϕ, the projection of r perpendicular to the line of sight is $r \sin \phi$. The average value of this projection is accordingly $\int r \sin \phi \, d\sigma / \int d\sigma$, the integration being extended over the whole sphere. Hubble, in his original paper, gave the value $\frac{2}{\pi}r$ for the average projection, but by the method just outlined we obtain the larger value $\frac{\pi}{4}r$, and accordingly a smaller correction.

Table III shows the comparison of the observational data with the theory. The second column contains the average observed value of $m+5 \log a_1$ taken from Hubble's table. An excess of this observed value over the value given by (13) means that the nebula is brighter than the intercepted starlight by this excess. In this way the empirical value of the ratio L has been obtained. Once this is known, the temperature T of the star can be calculated from Table II by interpolation. These values of T are given in the last column.

The temperatures for Bo and O stars associated with diffuse nebulosity calculated in this manner agree fairly well with those obtained from the theory of thermal ionization. R. H. Fowler and

TABLE III

Diffuse-Emission Nebulae	Observed Average $m+5 \log a_1$	Empirical Ratio L from (13)	Stellar Temp. T Calculated from Table II
O stars (10)	11.35	−0.26	34,000°
Bo stars (5)	10.37	0.72	28,000

E. A. Milne[1] give the approximate values Bo–O9, 26,500°; O5, more than 35,000°; Miss C. H. Payne[2] gives for O stars 35,000° (from observed He^+ lines) and, according to another method (observed Si^{+++}), 25,000°.

The values derived in the present paper are also approximate. Too much importance should not be placed on that calculated for the Bo stars, because of their small number. Also, two of them are of a peculiar type. Another, ζ Orionis, associated with the nebula N.G.C. 2024, should probably be discarded because of intervening nebulosity, as Dr. Hubble has informed me, and this would make the average temperature somewhat higher. On the other hand, if the color-excess of the remaining four stars is taken into account, and their observed magnitude corrected accordingly, the average temperature would be lowered by about the same amount. The empirical material for the O stars is much better, and, moreover, a correction on account of their color-excess would be insignificant.

[1] *Monthly Notices, R.A.S.*, **84**, 515, 1924.

[2] *Stellar Atmospheres*, p. 139.

Although not too much value should be attached to the observational material of the Bo stars, the fact that a lower temperature is found for them than is found for the O stars, as of course should be the case, is definitely in favor of the proposed mechanism.

No detailed discussion of observational data for planetary nebulae will be undertaken. However, the empirical material definitely indicates that the photographic intensity of the nebula may be greatly in excess of that of the intercepted starlight. In the planetary nebulae the "nebulium" spectrum of unknown origin becomes predominant, and hence they cannot be treated satisfactorily with the approximations that have been introduced. The significant fact remains, however, that the mechanism of ionization and recombination can account for a photographic nebular luminosity greatly in excess of that of the intercepted starlight, as is shown by Table II. Although no quantitative statement can be made at present, there is no doubt that the mechanism requires, on the average, a temperature of the central star of planetaries very much higher than 33,000°. This is in line with present evidence, e.g., from the distribution of intensity in the continuous spectrum of the central star.[1]

Mr. H. H. Plaskett has recently obtained the following interesting observational result which, through his courtesy, I am permitted to mention, though it has not yet been published. Investigating the intensity ratios of the lines of the Balmer series in one typical planetary and one typical diffuse nebula, he found that the decrease in intensity toward the higher lines of the series is much more rapid in the planetary than in the diffuse nebula. At the same time the temperature of the associated star in the planetary must be much the higher, since the line 4686 of ionized helium in this nebula is very strong. So it appears that in the typical case investigated by him, a higher temperature of the associated star is accompanied by a strengthening of the first lines of the Balmer series as compared with the higher lines. This fact seems to fit in quite well with the mechanism of luminosity proposed in the present discussion. According to it, the electrons freed photo-electrically by the ultra-

[1] See, for example, W. H. Wright, *Publications of the Lick Observatory*, **13**, 252, 253, 1918.

violet starlight are recaptured by the different levels of the hydrogen atom and fall back to the lowest level, either directly or in steps, thus producing line spectra. The more detailed physical picture made of this recapture implies that the probability of recapture by the lower levels becomes relatively larger, as the velocity of the free electron becomes higher. The higher the temperature of the star, therefore, the more electrons will be present at the lower levels after this recapture. But an electron on a lower level cannot possibly produce a line of the Balmer series originating on a higher level. Hence, as the temperature of the star increases, the capture of the electrons by the lower levels will result in a strengthening of the first lines of the Balmer series as compared with the higher lines and thus will account, in a qualitative way, for the observed phenomenon. Plaskett has pointed out that the distribution of intensity as observed by him could not be understood if the mechanism of luminosity were one of ordinary excitation, because in that case one would expect the lines requiring higher excitation to become relatively stronger as the temperature of the star increases. In the present discussion, the ordinary excitation by ultra-violet starlight was discarded as being totally inadequate to account for the observed intensity of luminosity. Mr. Plaskett's observation is additional and apparently conclusive evidence that the main mechanism causing the luminosity is one of ionization and recombination, and not that of ordinary excitation.

Certain theoretical considerations of H. A. Kramers[1] bear upon this interpretation. By an application of Bohr's correspondence principle Kramers derives an expression for the probability of capture of a free electron by the different energy-levels of a positive nucleus. The probability $Q_{i\to n}$ that a free electron of velocity v is captured in one second by the nth energy-level of the nucleus is given by the following formulae:

$$Q_{i\to n}=\frac{\beta v}{V}, \qquad \beta=\frac{K}{v^2 n^3 \nu} g', \qquad h\nu = W_n + \tfrac{1}{2} m v^2, \qquad W_n = hN\frac{1}{n^2},$$

which are Kramers' formulae (13), (56) or (60), and (11), and the formula for the ionization energy W_n of the nth level of the hydrogen

[1] *Philosophical Magazine*, **46**, 836, 1923.

atom. In those expressions, V is the volume considered; K, a constant expressible in terms of universal constants and the atomic number; h, Planck's constant; N, the Rydberg constant; and m, the mass of the electron. The quantity g' is equal to 1 in Kramers' formula (56) and approaches unity in (60), if $\frac{h}{e^2}v$ is sufficiently small (e = electronic charge).

Taking those formulae as they stand and putting $g' = 1$, which is strictly justified only for sufficiently small velocities v, we obtain for the fraction p_n of the total number of electrons which is captured by the nth level

$$p_n = \frac{\frac{1}{n+Cn^3}}{\sum_{n=1}^{n=\infty}\frac{1}{n+Cn^3}}, \qquad C = \frac{\frac{1}{2}mv^2}{W_1}.$$

The constant C is the ratio between the kinetic energy of the free electron and the ionizing energy, $W_1 = h\nu_0$, of the first level. If the velocity v of the electron and hence C approaches zero, the numerator approaches infinity, and p_n approaches zero for every level. This means that, considering any finite level n, the capture of electrons takes place in the higher levels up to infinity, and the contribution of a finite number of lower levels is negligible. On the other hand, if C is not zero, the numerator converges and, if the kinetic energy $\frac{1}{2}mv^2$ of the electron is comparable with W_1, an appreciable fraction will be captured by the first levels and this fraction drops off very rapidly, for the higher levels even as fast as $\frac{1}{n^3}$.

The foregoing indicates that the assumption that electrons with large velocities are to a great extent captured by the lower levels is in harmony with present theoretical views based on the correspondence principle.

E. A. Milne[1] has also discussed the capture of electrons, but not on the basis of the correspondence principle. For Kramers' quantity β (in Milne's notation $2\pi F/v$), which may be considered as an

[1] *Philosophical Magazine*, **47**, 209, 1924.

effective area of capture for a given energy-level, he derives an expression

$$\beta = \frac{\text{const.}}{v^2},$$

which, according to him, should hold for small velocities. Thus far Milne's and Kramers' results agree, as is seen from Kramers' expressions for β and $h\nu$, when v approaches zero. Milne, however, introduces the additional assumption that this expression holds for all velocities, from which would follow that the relative distribution of captured electrons on the different levels is independent of the velocity of the electron. It appears, therefore, that Milne's view is hard to reconcile with the observations of nebulae.

4. NEBULOSITY OF THE CONTINUOUS TYPE; TEMPERATURE OF A STAR B1

According to Hubble, stars of class B1 and later are usually connected with nebulosity emitting a continuous spectrum. There is hardly any doubt that the spectrum of such a nebula is produced by reflection of the starlight, and the formula (13) must be modified accordingly. If the nebular patch acts like a perfect reflector, the light emerging from the nebula is the equivalent of the intercepted starlight, but it is re-emitted only in a forward direction and not backward. Not aiming at an accurate statistical treatment, this may be taken into account by multiplying the surface luminosity of the nebula by 2, hence the appropriate formula is obtained by adding $2.5 \log 2 = 0.75$ to the right-hand side of equation (13). We have, therefore,

$$m + 5 \log a_1 = 11.84 , \qquad (14)$$

for the nebular patch acting as a perfect reflector. This formula, on the present interpretation, should hold for all nebulae of the continuous type.

The data for continuous nebulae are contained in Table I of Hubble's second paper, in which all stars of class B1 or later are assumed to produce a continuous spectrum, even if this has not been directly observed. The ten such stars of color-excess zero give an average value of $A = 10.73$ for $m + 5 \log a_1$. The other forty stars for

which the color-excess is known show a systematic discrepancy from this value. This evidently is due to detached patches of nebulosity which happen to be between the star and the observer. Such a patch of nebulosity absorbs and scatters the starlight, causing the observed m to be larger than the true m, and at the same time the scattering produces a positive color-excess. On this interpretation, one may assume that the true magnitude of the star is given by $m_t = m - k\epsilon$, where m is the observed magnitude and ϵ the color-excess, and therefore $(m - k\epsilon) + 5 \log a_1 = A$. A least-squares solution based on the data for all fifty stars gives the values $k = 1.4$ and $A = 10.61$. This means that a correction of -1.4 times the color-excess must be applied to the observed magnitude m in order to obtain the true magnitude m_t, and further that for stars producing a continuous nebular spectrum

$$m_t + 5 \log a_1 = 10.61 ,$$

on the average. This value of 10.61 for fifty stars is sufficiently close to that of 10.73 obtained for the ten stars with color-excess zero, and it should be the more accurate.[1]

If the light of the nebular patch is assumed to be re-emitted evenly in all directions, and is the equivalent of the starlight intercepted, the theoretical formula (13) holds. On this interpretation the empirical value 10.61 means that the continuous spectrum re-emitted by the nebular patch is less intense than the intercepted starlight by 0.48 mag., or that it is 64 per cent of the intercepted starlight. This is much the same inference as that of Hubble, who concluded that the light re-emitted was the equivalent of the light intercepted (sec. 1,*a*).

For continuous nebulae, however, the formula (14), based on the interpretation of reflection, appears to be preferable. The empirical value 10.61 means, then, that the reflection is not perfect, but that there is an albedo of $11.84 - 10.61 = 1.23$ mag., or about 30 per cent. The reflection theory leads, therefore, to a reasonable value of the albedo.

[1] Hubble, treating all the stars producing continuous or emission spectrum in the same way, obtained $k = 0.9$. We thought it better to leave out the stars producing emission spectrum, where the main cause of the difference in $m + 5 \log a_1$ appears to be the temperature of the star, and not the color-excess.

The continuous spectrum of a nebula under the influence of stars B1 or later is most likely due to reflection of the starlight by solid or liquid particles. If gases also are present, a faint nebular emission spectrum may be expected which, if sufficiently strong, will show up against the continuous background. The occurrence of both spectra together permits a rough value for the intensity of the emission spectrum to be obtained by matching it with the continuous spectrum, and thus leads to a theoretical value for the temperature of the associated star. The star B.D. $+46^{\circ}3474$, associated with the nebula I.C. 5146, is a case in point. Dr. Hubble has furnished the following unpublished description of the nebular spectrum. The photograph shows $H\beta$ and $H\gamma$ and a trace of $H\delta$ against a continuous background. The width of the first two lines on the photographic plate is about one-fortieth of the total spectrum. Their intensity appears to be of about the same order as that of the continuous background on which they are superimposed. According to this description, the intensity of the lines $H\beta$ and $H\gamma$ together may be roughly estimated as one-twentieth of that of the continuous spectrum. If twice this amount is allowed for the other lines of the Balmer series, the total intensity of the Balmer series would be roughly 15 per cent of that of the continuous spectrum. It has just been concluded that the continuous spectrum, which is re-emitted by the nebula in the forward direction only, contains about 30 per cent of the intercepted starlight. The part of the Balmer series which also is re-emitted forward is 15 per cent of this, but an equal amount is re-emitted backward, hence the total intensity of the Balmer series may be estimated to be roughly 2×15 per cent $\times 30$ per cent, or about one-tenth of the intercepted starlight, and hence $L=\frac{1}{10}$. According to Table II, this corresponds to a temperature of roughly $21{,}000^{\circ}$ for the associated star, B.D. $+46^{\circ}3474$, of type B1. Another case of a B1 star producing a nebular spectrum crossed by emission lines is c' Orionis, associated with the nebula N.G.C. 1977.[1] In three other cases of B1 stars where the nebular spectra have been observed (cometary nebula, I.C. 446 and B.D. $-12^{\circ}1771$) they have been listed as continuous. If observational conditions were about the same, we would expect for c' Orionis also a temperature of about

[1] V. M. Slipher, *Publications of the Astronomical Society of the Pacific*, **31**, 22, 1919.

21,000°. For the other three B1 stars it may be that the temperature is lower than 21,000°, or possibly that no gases are present.

Fowler and Milne, in their provisional temperature scale, give a temperature of roughly 19,000° for the stellar type B1. The agreement is as good as might be expected, the more so since B1 stars producing a nebular spectrum with emission lines are probably of a slightly earlier type than the normal B1.

For stars of a normal spectral type later than B1 and therefore of lower temperature, a continuous spectrum with emission lines would not be expected under the same observational conditions. This is in agreement with the observations as far as listed in Hubble's table. However, if a narrower slit could be used with longer exposure, the possibility is not excluded that for those stars also emission lines might be observed on the continuous background and a temperature estimate be carried out. The absence of emission lines, as previously remarked, does not necessarily mean a low temperature of the star, but if the emission lines are present, it appears probable that the absorption of the ultra-violet starlight is complete and that the line intensities can be used for temperature determinations.

Collecting the results, it is found that, as far as can be judged, the proposed mechanism of luminosity requires the following approximate values for the temperatures of stars associated with diffuse nebulosity: B1, 21,000° C. Abs.; B0, 28,000°; O, 34,000°; among which the value for B0 is probably not very reliable. Those values agree fairly well with the approximate temperatures obtained by Fowler and Milne: B1, 19,000°; B0, O9, 25,000°; O5, more than 35,000°.

The agreement of temperatures obtained from the luminosity of nebulae with those obtained from the theory of thermal ionization serves as a check for the mechanism of luminosity proposed. To this is added the further evidence of H. H. Plaskett's observations regarding the distribution of intensities in the Balmer series of the nebular spectrum.

Once convinced that the consequences of the proposed mechanism fit in fairly well with the present observations and theories, the temperatures which were obtained may be considered on their own merit. The luminosity of nebulae provides a method for obtaining

the temperatures of stars. An advantage of this method is that physical conditions in a nebula are very simple an account of the extremely low concentration of the nebular atoms. Certain points neglected in the present treatment may be taken into account when the physical theories of recombination and probability of transition become more reliable and more definite. The "nebulium" spectrum offers additional difficulties. The assumption that the ultra-violet starlight is practically complete may need further check. Hubble's statistical treatment of diffuse nebulae followed in this paper is based on a theoretical average value of $m+5 \log a_1$, which is only approximate. The greatest source of error, however, appears to arise from the difference in distribution of intensity of nebular spectrum and star spectrum, where the number of quanta between certain limits of frequency is introduced as an approximate measure of photographic intensity. This error may be removed by the introduction of better photographic methods. It is hard to give an estimate of the accuracy of the results, but it is believed that the total error may amount to 1 or even $1\frac{1}{2}$ mag., which corresponds to a rather wide range in temperature, according to the first and last column of Table II.

This work was begun at the University of Hamburg and finished at the California Institute of Technology. I am indebted to Dr. W. Pauli and Professor P. S. Epstein for help regarding the physical aspects of the problem. The problem itself was suggested to me by Dr. W. Baade, of the Bergedorf Observatory, who made me acquainted with its main features. The very active help of Dr. E. P. Hubble, of the Mount Wilson Observatory, gave me a more detailed knowledge of the observational material and its interpretation which led me to try out the mechanism of ionization and recombination as the fundamental cause of luminosity. Mr. H. H. Plaskett kindly allowed me to mention his unpublished observational result and to use it in this paper. In the course of the work it was discussed with many persons, in particular Dr. R. Minkowski, Dr. S. Rosseland, Professor H. N. Russell, and Dr. I. S. Bowen.

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REVIEWS

Beobachtungen der Dämmerung und von Ringerscheinungen um die Sonne, 1911 bis 1917. By C. DORNO. (*Veröffentlichungen des Königlich Preussischen Meteorologischen Instituts,* No. 295). Berlin: Behrend & Co., 1917. 4to. Pp. 94. 11 figs. Paper. 9 M.

Himmelshelligkeit, Himmelspolarization und Sonnenintensität in Davos, 1911 bis 1918. By C. DORNO. (*Veröffentlichungen des Preussischen Meteorologischen Instituts,* No. 303). Berlin: Behrend & Co., 1919. 4to. Pp. 290. 26 figs. and 68 tables. Paper. 54 M.

The first of these publications deals with the appearance and disappearance of the colors at sunset and sunrise, and with atmospheric halos, coronas, and light pillars about the sun. Naked-eye observations, of course, predominate in this work. In addition, a Jensen pendulum and quadrant was used for altitudes, a theodolite for azimuths, and a L. Weber polarizing photometer for measurements of intensities. The author must have gotten much pleasure from observing these phenomena, which are among the most beautiful in all nature. In addition, he has studied his observations and has compared them with such theories of the physics of the air as exist.

The second publication is very ambitious and discusses a great variety of observations in almost bewildering detail. Hardly anything more than an enumeration of the more important lines of observation can be given in this brief notice.

The total illumination of a plane surface by sun and sky separately was measured with the aid of Weber's polarizing photometer arranged for photography.

Illumination by delimited areas of the sky at varying distances from the sun, with the sun at different altitudes throughout the day and the year, was studied. For color photometry with light filters the photoelectric method was used. The author's observations cover the period, 1912–1914, of great atmospheric turbidity caused by the eruption of Katmai.

Polariscopic observations of light from such areas under all possible conditions of the sky were taken.

Determination of the total radiation of the sun received at Davos, Switzerland, elevation, 1600 meters above the sea, as well as of the partial radiation transmitted through light filters of several kinds, was made. An Ångstrom pyrheliometer and W. Michelson's actinometer were added to the other equipment for this part of the work. Dr. Dorno is deeply impressed with the difficulty of evaluating the corrections for atmospheric absorption and scattering of solar rays, and admits that he has not gone into the matter exhaustively.

Dr. Dorno's treatment is analytic, and scholarly in his consistent practice of comparing his own findings with those of others and with theory. Full references are given throughout. The style is distinctly verbose—a quality which often must be condoned, especially when the subject of study has so many ever present variables as has meteorology. It is a source book for the student of meteorology.

There are complete tables of contents for text, tables, and figures, but there is no general index, either of subjects or of names.

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